QP CODE: 22101057

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Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS, APRIL 2022

Sixth Semester

CORE - MM6CRT01 - REAL ANALYSIS

Common for B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science & B.Sc Computer Applications Model III Triple Main

2017 Admission Onwards

7FF02C27

Time: 3 Hours

Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Prove that the signum function is not continuous at 0.
2. Let f be defind for all $x \in R, x \neq 2$ by $f(x) = \frac{x^2+x-6}{x-2}$. Define f at x = 2 in such a way that f is continuous at that point.
3. Show that the continuous image of an open interval need not be an open interval.
4. Determine whether the function $f(x) = x|x|, \forall x$ is differentiable and find its derivative?
5. Given that the function $f : R \to R$ defined by $f(x) = x^5 + 4x + 3$ is invertible and let g be its inverse. Find the value of $g'(8)$?
6. Prove that a function $f : I \to R$ is decreasing if $f'(x) \leq 0, \forall x \in I$. Where $f'(x)$ denote the derivative of the function?
7. Define a step function.
8. Let $f : [a, b] \to \mathbb{R}$ and $C \in \mathbb{R}$. Show that if ϕ is an antiderivative of $fon[a, b]$ then $\phi + C$ is also an antiderivative of f on $[a, b]$
9. State any theorem which characterises Riemann Integrable function on an interval $[a, b]$.
10. Define pointwise convergence of a sequence of functions with example.
11. Define uniform convergence of a sequence of functions with example.
12. Show that the sequence $\left(\frac{x^n}{1+x^n}\right)$ does not converge uniformly on $[0, 2]$ by showing that the limit function is not continuous on $[0, 2]$.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Prove or disprove:" *If I = [a, b] and* $f : I \to R$ *is continuous on I then* $f(I) = [f(a), f(b)]$."

14. Show that the function $f(x) = \frac{1}{x}$ is uniformly continuous on the set $A = [a, \infty)$,where a is a positive constant.

15. Define Jump at c of the function $f : [a, b] \to R$ where a < c < b and show that f is continuous at c iff $J_f(c) = 0$.

16. State and prove the product rule of differentiation?

17. State and prove L'Hospitals Rule II?

18. Evaluate the limit $\lim_{x \to 0+} (x)^{\sin x}, x \in (0, \infty)$

19. Let $f(x) = 1, for x = \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}.$ and $f(x) = 0, elsewhere in [0, 1]$, show that $f \in \mathcal{R}[0, 1]$ and $\int\limits_0^1 f = 0.$

20. Evaluate $\int\limits_1^4 \frac{cos\sqrt{t}}{\sqrt{t}} dt.$

21. Let $g_n : [0, 1] \to \mathbb{R}$ defined by $g_n(x) = x^n$. Show that (g_n) converges but the limit is not differentiable on $[0, 1]$.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. (a) Let I = [a,b] be a closed bounded interval and let $f : I \to R$ be continuous on I. Then prove that f has an absolute maximum and an absolute minimum on I .
(b) State and prove Preservation of intervals Theorem.

23. 1. State and prove the Mean Value theorem?
2. Using Mean value theorem, Prove the following inequalities
(a.) $e^x \geq 1 + x, \forall x \in R$
(b.) $-x \leq \sin x \leq x, \forall x \geq 0$

24. Let $[a, b]$ be an interval in $\mathbb{R}$ and let $\mathcal{R}[a, b]$, $C'[a, b]$ and $C[a, b]$ denotes set of all Riemann integrable, st of all differentiable and set of all continous real valued functions on $[a, b]$ respectively.
(a) Show that $\mathcal{R}[a, b]$ is a vector space over the filed of real numbers $\mathbb{R}$.

(b). Show that $C'[a,b]$ is a **proper** subspace of$C[a,b]$ and $C[a,b]$ is **proper** subspace of $\mathcal{R}[a,b]$.

25. (a) State and prove the Cauchy Criterion for Riemann integrability of a function $f:[a,b]\to\mathbb{R}$.
(b) Check the Riemann integrability of Dirichlet function.

(2×15=30)