QP CODE: 21101239

21101239

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) EXAMINATION, APRIL 2021

Sixth Semester

CORE - MM6CRT03 - COMPLEX ANALYSIS

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

2846D059

Time: 3 Hours | Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Find the domain of definition of f(z)= $\frac{1}{1-|z|^2}$
2. Show that f(z)= $\bar{z}$ is no where differentiable.
3. Prove that if the real part of an analytic function is constant,then the function is constant
4. Prove that Log $(1+i)^2$= 2Log (1+i),but Log $(-1+i)^2 \neq$ 2Log (-1+i)
5. Prove that e^{iz}=cos z+i sin z
6. Evaluate $\int_1^2 (\frac{1}{t} - i)^2 dt.$
7. Evaluate $\int_C ze^{-z} dz$ where C is the circle |z|=1.
8. Define simply connected and multiply connected domain.
9. Show that when $z \neq 0,\ \frac{\sin(z^2)}{z^4} = \frac{1}{z^2} - \frac{z^2}{3!} + \frac{z^6}{5!} - \frac{z^(10)}{7!} + \ldots$, assuming a series expansion of $\sin z$
10. Obtain a Maclaurin series expansion of z^2e^{3z}
11. State a necessary and sufficient condition for an isolated singular point z_0 of a function $f(z)$ to be a pole of order m. Also give the formula for the residue at z_0.
12. Prove that if the improper integral over $-\infty < x < \infty$ exists, then its Cauchy Principal Value exists.

(10×2=20)

Turn Over

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Show that the function defined by $f(z) = \begin{cases} \frac{(x+y)^2}{x^2+y^2} & \text{if } z \neq 0 \\ 1 & \text{if } z = 0 \end{cases}$ is discontinuous at the origin.

14. Prove that |exp(-2z)|<1 if and only if Re(z)>0

15. Find where $\tan^{-1} z = \frac{i}{2} \, \log \frac{i+z}{i-z}$ is analytic.

16. Evaluate $\int_C \frac{z+2}{z} dz$, where C is the semicircle $z = 2e^{i\theta}, \quad (\pi \leq \theta \leq 2\pi)$.

17. Prove that a function f is analytic at a given point, then its derivative of all orders are analytic at that point.

18. Prove that any polynomial of degree n has atleast one zero.

19. Give two Laurent series expansions in powers of z for the function $f(z) = \frac{1}{z^2(1-z)}$ and specify the regions in which those expansions are valid.

20. Using residues, evaluate $\displaystyle\int_C \frac{1}{z(z-2)^2} dz$, where C is the unit circle $|z-2| = 1$

21. State and prove Cauchy's Residue Theorem.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. Prove that the function u(x,y)=e^x(x siny+y cosy) is harmonic and determine in terms of z,the most general analytic functon f(z) with real part u(x,y)

23.
- State and Prove Cauchy's Integral Formula.
- Evaluate $\displaystyle\int_{|z|=1} \frac{\cos z}{z(z-4)} dz$

24. a) State and prove a necessary and sufficient condition for convergence of sequence $z_n = x_n + iy_n$ of complex numbers.
b) Using this derive a necessary and sufficient condition for convergence of series $\displaystyle\sum_{n=1}^{\infty} z_n$ of complex numbers, where $z_n = x_n + iy_n$.

c) Prove that if a series of complex numbers converges, then the n^{th} term converges to zero, as n tends to infinity.

25. Explain the three types of isolated singular points of a complex function with examples. Verify the examples with their series representations.

(2×15=30)