21101238

QP CODE: 21101238

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) EXAMINATION, APRIL 2021

Sixth Semester

CORE - MM6CRT02 - GRAPH THEORY AND METRIC SPACES

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

9FAF2725

Time: 3 Hours | Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Define a Graph. Define a loop in a graph.
2. When will you say that two graphs are isomorphic?
3. Draw all non-isomorphic complete bipartite graphs with atmost 4 vertices.
4. Define a walk. When will you say that a walk is open?
5. Define a tree. Draw a tree which is a complete graph.
6. Define spanning trees. How many spanning trees are there for K4?
7. Define Eulerian graph. Is K3 Eulerian? Justify.
8. Define closure of a graph . Draw one example.
9. Define metric space.
10. Let (X,d) be a metric space and $A \subseteq X$. Define an interior point of A.
11. Define convergence in a metric space using metric.
12. Define isometry.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Let G be a simple graph with n vertices and let $\bar{G}$ be its complement. Prove that for each vertex v in G, $d_G(v)+d_{\bar{G}}(v) = n-1$.
14. Define incidence matrix of a graph.What can you say about the sum of the elements in the i^{th} row of of the incidence matrix of the graph. Write down the incidence marix of K_4

15. If G be a graph with n vertices and q edges. Let $\omega(G)$ denote the number of connected components of G. Then prove that G has at least $n - \omega(G)$ edges.

16. a) Define cut vertex of a graph.
b) Let v be a vertex of the connected graph G . Then prove that 'v' is cut vertex of G if and only if there are two vertices 'u' and 'w' of G , both different from 'v', such that 'v' is on every u – w path in G.

17. Prove that a simple graph G is Hamiltonian if and only if its closure C(G) is Hamiltonian.

18. Prove that in any metric space X, the empty set ø and the full space X are open sets.

19. Define Cantor set. Prove that there exist infinitely many points in Cantor set.

20. Let X be a metric space. If $\{x_n\}$ and $\{y_n\}$ are sequences in X such that $x_n \to x$ and $y_n \to y$, show that $d(x_n, y_n) \to d(x, y)$.

21. If $\{A_n\}$ is a sequence of nowhere dense sets in a complete metric space X, then prove that there exists a point in X which is not in any of the $A_n' s$.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22.
(a)State and prove First theorem of graph theory.
(b)Prove that in any graph G there is an even number of odd vertices.
(c)Let G be a k-regular graph, where k is an odd number. Prove that the number of edges in G is a multiple of k.

23. a)Let G be simple graph with at least three vertices. Then prove that G is 2- connected if and only if for each pair of distinct vertices u and v of G , there are two internally disjoint u – v paths in G.
b) Let u and v be two vertices of the 2- connected graph. Then prove that there is a cycle passing through both u and v.

24. a) In any metric space X prove that the empty set ø and the full set X are closed sets.
b) Prove that a subset F of a metric space X is closed if and only if its complement F' is open.

25. a) Prove that if a convergent sequence in a metric space has infinitely many distinct points, then its limit is a limit point of the set of points of the sequence.
b) Will the result be true, if the condition infinitely many distinct points is not given? Justify.

(2×15=30)