20100570

QP CODE: 20100570

Reg No : ....

Name : ....

BSc DEGREE (CBCS) EXAMINATION, MARCH 2020

Sixth Semester

Core course - MM6CRT04 - LINEAR ALGEBRA

B Sc Mathematics Model I,B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

FEF9B611

Time: 3 Hours

Marks: 80

Part A

Answer any ten questions.

Each question carries 2 marks.

1. Prove that every mxn matrix A there is a unique mxn matrix B such that A+B = 0
2. Define an orthogonal matrix. Give an example of an orthogonal matrix.
3. a) Define an invertible matrix
   b)Prove that if A is invertible then$(A^{-1})' = (A')^{-1}$
4. Define a basis of a vector space V and Prove that { (1,1), (1,-1)} is a basis of R2.
5. Define dimension of a vector space V and Find the dimension of Rn [X]
6. Define departure space and arrival space of a linear mapping. Give an example.
7. Define linear isomorphism of vector spaces Give an example.
8. Define an ordered basis of a vector space. Prove that every basis of n elements give rise to $n!$ distinct ordered bases.
9. Define transition matrix from the basis $(v_i)_m$ to the basis $(v_i')_m$ of a vector space V.
10. If λ is an eigen value of an invertible matrix A, then prove that $\lambda \neq 0$ and λ^{-1} is an eigen value of A^{-1}.
11. Define eigen value of a linear map and the eigen vector associated with it.
12. Define diagonalizable linear map and diagonalizable matrix.

(10×2=20)

Part B

Answer any six questions.

Each question carries 5 marks.

13. Reduce the following matrix to row echelon form $\begin{bmatrix} 1 & 2 & 0 & 3 & 1 \\ 1 & 2 & 3 & 3 & 3 \\ 1 & 0 & 1 & 1 & 3 \\ 1 & 1 & 1 & 2 & 1 \end{bmatrix}$

14. Find the row rank of the matrix $\begin{bmatrix} 1 & 2 & 5 \\ 2 & 1 & 4 \\ 0 & -1 & -2 \\ 0 & 1 & 2 \end{bmatrix}$

15. Prove that Mmxn (R) be the set of all mxn matrices is a vector space

16. Prove that the intersection of any set of subspaces of a vector space V is a subspace of V

17. Define injective linear mapping. Prove that if the linear mapping $f : V \to W$ is injective and $\{v_1, v_2, \ldots, v_n\}$ is a linearly independent subset of V then $\{f(v_1), f(v_2), \ldots, f(v_n)\}$ is a linearly independent subset of W.

18. a) Define rank and nullity of a linear mapping. Find the rank and nullity of $pr_1 : \mathbb{R}^3 \to \mathbb{R}$ defined by $pr_1(x, y, z) = x$.
b) Let V and W be vector spaces each of dimension n over a field F. If $f : V \to W$ is linear, then prove that f is surjective if and only if f is bijective.

19. a) Define a nilpotent linear mapping f on a vector space V of dimension n over a field F. What is meant by index of nilpotency of f.
b) Suppose that f is nilpotent of index p. If $x \in V$ is such that $f^{p-1}(x) \neq 0$, prove that $\{x, f(x), f^2(x), \ldots, f^{p-1}(x)\}$ is linearly independent

20. Find the eigen values and their algebraic multiplicity of $\begin{bmatrix} -3 & 1 & -1 \\ -7 & 5 & -1 \\ -6 & 6 & -2 \end{bmatrix}$

21. For the nXn tridiagonal matrix An = $\begin{bmatrix} 2 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 2 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 2 & 1 & \cdots & 0 & 0 \\ . & . & . & . & \cdots & . & . \\ 0 & 0 & 0 & 0 & \cdots & 2 & 1 \\ 0 & 0 & 0 & 0 & \cdots & 1 & 2 \end{bmatrix}$ Prove that det An = n + 1.

(6×5=30)

Part C

Answer any two questions

Each question carries 15 marks.

22 a)Prove that if A is an mxn matrix then the homogeneous system of equation $Ax = 0$ has a nontrivial solution if and only if rank $A < n$

b)Show that the matrix $A = \begin{bmatrix} 1 & 2 & -1 & -2 \\ -1 & -1 & 1 & 1 \\ 0 & 1 & 2 & 1 \end{bmatrix}$ is of rank 3 and final matrices P,Q such that $PAQ = [I3, 0]$.

c)Show that the system of equations $x + y + z + t = 4$, $x + \beta y + z + t = 4$, $x + y + \beta z + (3 - \beta)t = 6$, $2x + 2y + 2z + \beta t = 6$ has a unique solution if $\beta \neq 1, 2$.

23 *a) Prove that $P(x) = 2 + x + x^2$, $q(x) = x + 2x^2$, $r(x) = 2 + 2x + 3x^2$ is linearly dependent*

b)Let S_1 and S_2 be non empty subsets of a vector space such that $S_1 \subseteq S_2$. Prove that

1) If S_2 is linearly independent then S_1 is also linearly independent

2) If S_1 is linearly dependent then S_2 is also linearly dependent.

c)Determine which of following subsets of M_{3x1} R are linearly dependent

i) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \right\}$ ii) $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \right\}$

24 a) Define $Im\ f$ and $Ker\ f$ where f is a linear mapping from a vector space to a vector space.

b) Write image and kernel for $f_A : Mat_{n\times1}\,\mathbb{R} \to Mat_{n\times1}\,\mathbb{R}$ described by $f_A(\mathbf{x}) = A\mathbf{x}$ where A is a given real $n \times n$ matrix.

c) Find $Im\ f$ and $Ker\ f$ when $f : \mathbb{R}^3 \to \mathbb{R}^3$ is given by $f(a, b, c) = (a + b,\ b + c,\ a + c)$.

25 a) Define similar matrices and state whether similar matrices have the same rank. Show that if matrices A, B are similar then so are A', B'.

b) Check whether for every $\vartheta \in \mathbb{R}$, the complex matrices $\begin{bmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{bmatrix}$, $\begin{bmatrix} e^{i\vartheta} & 0 \\ 0 & e^{-i\vartheta} \end{bmatrix}$ are similar.

c) Prove that the relation of being similar is an equivalence relation on the set of $n \times n$ matrices.

(2×15=30)