20100569

QP CODE: 20100569

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Reg No : ....................

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# BSc DEGREE (CBCS) EXAMINATION, MARCH 2020

## Sixth Semester

## Core course - MM6CRT03 - COMPLEX ANALYSIS

B.Sc Mathematics Model I,B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

2A6911B0

Time: 3 Hours

Marks: 80

### Part A

*Answer any ten questions.*

*Each question carries 2 marks.*

1. Find f '(z) where f(z) =z Im z
2. Find the singular points of the function f(z)=$\frac{z^3+7}{z^2-5z+6}$
3. Find the real part of $e^{-3z}$?
4. Find $i^{-2i}$.
5. Define the hyperbolic sine and hyperbolic cosine of a complex variable z
6. Evaluate $\int_1^2 (\frac{1}{t} - i)^2 dt$.
7. State Cauchy-Goursat Theorem.
8. Evaluate $\int_C \frac{e^z}{z-2} dz$, C is the circle |z|=3.
9. Define the convergence of an infinite series of complex numbers.
10. Derive the Maclaurin series expansion for $f(z) = cosz$, using the definition of $cosz = \frac{e^{iz}-e^{-iz}}{2}$
11. Find the residue at $z = 0$ of $f(z) = zcos(\frac{1}{z})$
12. Define removable singularity of a point $f(z)$. Why it is called so?

(10×2=20)



Turn Over

## Part B

*Answer any six questions.*

*Each question carries 5 marks.*

13. Express the function f(z)=$x^2$-$y^2$ -2y+i(2x-2xy) where z=x+iy in terms of z

14. Let f$f(z) = \begin{cases} \frac{\bar{z}^3}{z^2} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$

Prove that

a) f(z) is continuous everywhere on C

b) The complex derivative $f'(0)$ does not exist

15. Find an analytic function f(z) in terms of z and with real part $u = y - \frac{1}{2}y^2 + \frac{1}{2}x^2$

16. Evaluate $\int_C \frac{z+2}{z} dz$, where C is the semicircle $z = 2e^{i\theta}$, $(0 \leq \theta \leq \pi)$.

17. State and prove Cauchy's inequality.

18. State and prove Fundamental theorem of Algebra

19. Assuming a series expansion of $e^z$,*show that* $z \cosh z^2 = \sum_{n=0}^{\infty} \frac{z^{4n+1}}{(2n)!} dz, |z| < \infty$

20. State a necessary and sufficient condition for an isolated singular point $z_0$ of a function $f(z)$to be a pole of order $m$ and the formula for residue at $z_0$ of $f(z)$ . Find the residue at $z = 3i$ of $f(z) = \frac{z+1}{z^2+9}$.

21. Define the improper integral of $f(x)$over $-\infty < x < \infty$ and its Cauchy Principal Value. Show that the existence of Cauchy Principal Value does not imply the existence of $\int_{-\infty}^{\infty} f(x)dx$ .

(6×5=30)

## Part C

*Answer any two questions.*

*Each question carries 15 marks.*

22. Prove that 1) $\sin^{-1} z = -i[\log iz + (1 - z^2)^{\frac{1}{2}}]$ .Hence deduce $\tan^{-1}z$

2) Evaluate $\tan^{-1}(1+i)$

23.
- State and Prove Cauchy's Integral formula.
- Find the value of $\int_C \frac{1}{(z^2+4)^2} dz$, where C is the circle $|z - i| = 2$ in the positive sense.

24. *a)* Derive the Laurent series expansion of $\frac{e^z}{(z+1)^2}$ in terms of $z+1$, *if* $0 < |z+1| < \infty$

*b)* *Let* $f(z) = \frac{1}{(z-i)^2}$ . Use Laurent series expansion to prove that $\int_C \frac{dz}{(z-i)^{-n+3}} = 2\pi i, n = 2$

*c)* Show that for $0 < |z-1| < 2$ $\frac{z}{(z-1)(z-2)} = \frac{-1}{2(z-1)} - 3\sum_{n=0}^{\infty} \frac{(z-1)^n}{2^{n+1}}$

25. State and prove Cauchy's Residue Theorem. Using the theorem, evaluate $\int_C \frac{5z-2}{z(z-1)} dz$ , where C is the circle $|z| = 2$

(2×15=30)