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20100568

QP CODE: 20100568

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Reg No : ....

Name : ....

BSc DEGREE (CBCS) EXAMINATION, MARCH 2020

Sixth Semester

Core course - MM6CRT02 - GRAPH THEORY AND METRIC SPACES

B.Sc Mathematics Model I,B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

4A7743A4

Time: 3 Hours

Maximum Marks :80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Define edge set of a graph. When will you say that two edges are parallel in a graph?
2. Give two different drawings of $K_{3,3}$ which are isomorphic.
3. Define a complete bipartite graph. Give an example.
4. Define an edge deleted subgraph.
5. Define a tree. Draw all non - isomorphic trees with 5 vertices.
6. Define Cut vertex and Draw one example.
7. Define an Euler tour of a graph G and an Euler graph.
8. Define Hamiltonian graph. Is K_4 Hamiltonian, justify.
9. Define usual metric on **R**.
10. Define closed set in a metric space (X,d).
11. Give an example of a sequence which is convergent, but the underlying set do not have a limit point.
12. Let X and Y be metric spaces and f a mapping of X into Y. If $x_n \to x_0$ implies $f(x_n) \to f(x_0)$ then prove that f is continuous at x_0.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Define eccentricity, diameter and radius of a connected graph G with vertex set V. Which simple graphs have diameter 1?

Turn Over

14. Define adjacency matrix of a graph. Find the graph whose adjacency matrix is $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

What can you say about the graph if all the entries of the main diagonal are zero?

15. Prove that an edge 'e' of a graph G is a bridge if and only if 'e' is not a part of any cycle in G .

16. Prove that a graph G is connected if and only if it has a spanning tree.

17. Let G be a simple graph with n vertices and let u and v be non-adjacent vertices in G such that $d(u) + d(v) \geq n$. Then prove that G is Hamiltonian if and only if G + uv is Hamiltonian.

18. Prove that every open sphere is an open set.

19. Write a short note on boundary of a set.

20. Define Cauchy sequence in a metric space. Prove that every convergent sequence is Cauchy. Give an example of a sequence which is Cauchy, but not convergent.

21. Let X be a complete metric space and let Y be a subspace of X. Prove that Y is complete if and only if it is closed.

(6×5=30)

Part C

Answer any ***two*** *questions.*

Each question carries ***15*** *marks.*

22. (a) State and prove First theorem of graph theory.
(b) Prove that in any graph G there is an even number of odd vertices.
(c) What is the smallest number of n such that the complete graph K_n has atleast 500 edges?

23. a) State and prove Whitney's theorem for 2- connected graphs.
b) Let u and v be two vertices of the 2- connected graph. Then prove that there is a cycle passing through both u and v.

24. a) Let A and B be two subsets of a metric space X, then prove or disprove
(i) int(A∩B) = int(A) ∩ int(B) (ii) int(A∪B) = int A ∪ int B
b) Prove that int a is the union of all open sets in A.
c) Prove that A is open if and only if A = int A.

25. (a) If $\{A_n\}$ is a sequence of nowhere dense sets in a complete metric space X, then prove that there exists a point in X which is not in any of the A_n's.
(b) State Baire's theorem. Explain how it is related to the above result.

(2×15=30)