QP CODE: 23004832

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Reg No : ....

Name : ....

MSc DEGREE (CSS) EXAMINATION , JULY 2023

Second Semester

CORE - ME010204 - COMPLEX ANALYSIS

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 Admission Onwards

0B485252

Time: 3 Hours | Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. What is Riemann Sphere?
2. Define Cauchy sequence. Show that every convergent sequence is a Cauchy sequence.
3. Evaluate $\int_{|z|=1} |z-1||dz|$.
4. Prove that $n(-\gamma, a) = -n(\gamma, a)$ where $n(\gamma, a)$ is the index of the point 'a' with respect to the closed curve γ.
5. Compute $\int_{|z|=1} \frac{e^z}{z} dz$
6. State the Cauchy's integral formula for higher derivatives. Evaluate $\int\limits_{|z|=1} e^z z^{-n} dz$.
7. Let $f(z)$ be analytic in a region Ω containing the point a. Prove that the function $F(z) = \frac{f(z)-f(a)}{z-a}$ has a removable singularity at $z = a$.
8. Prove that the zeros of an analytic function cannot have a limit point.
9. Define a locally exact differential.
10. State the argument principle.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Prove that an analytic function in a region Ω whose derivative vanishes identically must reduce to a constant. Also prove that the same is true if either the real part or the imaginary part is a constant.

12. Show that a linear transformation $w = \frac{az+b}{cz+d}$, c ≠ 0 is composed by a translation, an inversion , a rotation and a homothetic transformation.

13. Define rectifiable arcs. State and prove the necessary and sufficient condition for an arc to be rectifiable.

14. State and prove Cauchy's theorem for a rectangle with exceptional points.

15. State and prove the Weirstrass's theorem for essential singularities.

16. State and prove the maximum principle.

17. Explain the method for calculation of residues.

18. Evaluate $\int_0^\infty \frac{xsinmxdx}{x^2+a^2}, a > 0.$

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. (i) Reflect the imaginary axis, the line $x = y$ and the circle $|z| = 1$ in the circle $|z-2| = 1$.
(ii) State and prove the symmetry principle for linear transformation

20. State and prove Cauchy's theorem in a disk with exceptional points.

21. (a) Prove that a non constant analytic function maps open sets onto open sets.
(b) If $f(z)$ is analytic with $f'(z_0) \neq 0$, prove that it maps a neighborhood of z_0 conformally and topologically onto a region.

22. *Prove that* If $f(z)$ is analytic in Ω, then $\int_\gamma f(z)dz = 0$ for all cycles γ is homologues to zero in Ω .

(2×5=10 weightage)