24001059

QP CODE: 24001059

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS, MARCH 2024

Sixth Semester

CORE COURSE - MM6CRT04 - LINEAR ALGEBRA

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

6A418A62

Time: 3 Hours

Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Define the Hermite matrix. Give an example of a Hermite matrix.

2. a)Define linearly dependent rows.

   b)*Prove that in the matrix* $A = \begin{bmatrix} 1 & 2 & 0 & 2 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ *the columns are linearly dependent.*

3. If V is a Vector space over a field F. Prove that a) $\forall \lambda \in F, \lambda 0 = 0$ b) $\forall x \in V, 0x = 0$

4. *Prove that {(x, y, z, t) : x = y, z = t} is a subspace of* R^2

5. Check whether { (1,1,2), (1,2,5), (5,3,4) } is a basis of R3.

6. If $f : V \to W$ is linear, X is a subset of V and Y is a subset of W, define direct image of X under f and inverse image of Y under f.

7. Determine the transition matrix from the ordered basis $\{(1,0,0,1),(0,0,0,1),(1,1,0,0),(0,1,1,0)\}$ of $\mathbb{R}^4$ to the natural ordered basis of $\mathbb{R}^4$.

8. a) Define similar matrices.

   b) "Similar matrices have the same rank"-True or False?

9. Define a nilpotent linear mapping f on a vector space V of dimension n over a field F. What is meant by index of nilpotency of f.

10. Find the eigen values of A = $\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$

11. Define eigen value of a linear map and the eigen vector associated with it.

12. Define diagonalizable linear map and diagonalizable matrix.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. a) Prove that addition of matrices is associative.

b) Write 3x3 matrix whose entries are given by $x_{ij} = (-1)^{i-j}$

14. a) If A and B are orthogonal nxn matrices prove that AB is orthogonal.

b) Prove that a real 2x2 matrix is orthogonal if and only if it is of one of the forms $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$,

$\begin{bmatrix} a & b \\ b & -a \end{bmatrix}$ Where $a^2 + b^2 = 1$.

15. *a)Define span S of a vector space V and Prove that S = {(1,0), (0,1)} is a spanning set of R^2*

b)Prove that { (1,1,0), (2,5,3), (0,1,1)} of R^3 is linearly dependent.

16. If S is a subset of V, then prove that S is a basis if and only if S is a maximal independent subset.

17. Define $Im\, f$ and $Ker\, f$ where f is a linear mapping from a vector space to a vector space. Write image and kernel for the i-th projection of $\mathbb{R}^n$ onto $\mathbb{R}$.

18. Define injective linear mapping. Prove that if the linear mapping $f : V \to W$ is injective and $\{v_1, v_2, \ldots, v_n\}$ is a linearly independent subset of V then $\{f(v_1), f(v_2), \ldots, f(v_n)\}$ is a linearly independent subset of W.

19. a) Let V be a vector space of dimension $n \geq 1$ over a field F. Then prove that V is isomorphic to the vector space F^n.

b) If V and W are vector spaces of the same dimension n over F, then prove that V and W are isomorphic.

20. Determine the eigen values and their algebraic multiplicities of the linear mapping f: R3 → R3 given by f(x,y,z) = (x+ 2y + 2z, 2y + z, -x + 2y+ 2z)

21. For the nXn tridiagonal matrix $A_n = \begin{bmatrix} 2 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 2 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 2 & 1 & \cdots & 0 & 0 \\ . & . & . & . & \cdots & . & . \\ 0 & 0 & 0 & 0 & \cdots & 2 & 1 \\ 0 & 0 & 0 & 0 & \cdots & 1 & 2 \end{bmatrix}$ Prove that det $A_n = n + 1$.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. a) Prove that if A is an mxn matrix then the homogeneous system of equation Ax = 0 has a nontrivial solution if and only if rank A < n.

b) Show that the matrix $A = \begin{bmatrix} 1 & 2 & -1 & -2 \\ -1 & -1 & 1 & 1 \\ 0 & 1 & 2 & 1 \end{bmatrix}$ is of rank 3 and final matrices P,Q such that $PAQ = [I_3, 0]$.

c) Show that the system of equations $x + y + z + t = 4$, $x + \beta y + z + t = 4$, $x + y + \beta z + (3 - \beta)t = 6$, $2x + 2y + 2z + \beta t = 6$.has a unique solution if $\beta \neq 1, 2$.

23. a) Define a left inverse and right inverse of a matrix.

b) Prove that the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 3 & 3 \end{bmatrix}$ has a common unique left inverse and unique right inverse.

c) Find the inverse of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix}$

d) If $A_1, A_2, \ldots, A_P$ are invertible nxn matrices, prove that the product $A_1 A_2 \ldots A_P$ is invertible and that $(A_1 A_2 \ldots A_P)^{-1} = A_P^{-1} \ldots A_2^{-1} A_1^{-1}$

24. Let V and W be vector spaces each of dimension n over a field F. If $f : V \to W$ is linear then prove that the following statements are equivalent:
(i) f is injective (ii) f is surjective (iii) f is bijective (iv) f carries bases to bases, in the sense that if $\{v_1, \ldots, v_n\}$ is a basis of V then $\{f(v_1), \ldots, f(v_n)\}$ is a basis of W.

25. A linear mapping $f : \mathbb{R}^3 \to \mathbb{R}^3$ is such that $f(1,0,0) = (0,0,1)$, $f(1,1,0) = (0,1,1)$, $f(1,1,1) = (1,1,1)$. Determine $f(x, y, z)$ for all $(x, y, z) \in \mathbb{R}^3$ and compute the matrix of f relative to the ordered basis $B = \{(1,2,0),(2,1,0),(0,2,1)\}$. If $g : \mathbb{R}^3 \to \mathbb{R}^3$ is the linear mapping given by $g(x, y, z) = (2x,\ y + z,\ -x)$, compute the matrix $f \circ g \circ f$ relative to the ordered basis B.

(2×15=30)