QP CODE: 24001052

24001052

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS, MARCH 2024

Sixth Semester

CORE COURSE - MM6CRT01 - REAL ANALYSIS

Common for B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science & B.Sc Computer Applications Model III Triple Main

2017 Admission Onwards

25168F10

Time: 3 Hours

Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Let f be defind for all $x \in R, x \neq 2$ by $f(x) = \frac{x^2+x-6}{x-2}$. Define f at x = 2 in such a way that f is continuous at that point.

2. Give an example of a function $f : [0,1] \to R$ that is discontinuous at every point of [0,1] but such that |f| is continuous on [0,1].

3. Define absolute maximum point and absolute minimum point for $f : A \to R$.

4. Is every continuous function differentiable? Justify with proper reasoning or counter example.

5. Given that the function $f : R \to R$ defined by $f(x) = x^3 + 2x + 1$ is invertible and let g be its inverse. Find the value of $g'(1)$.

6. Define decreasing function with a proper example.

7. Define norm of the partition of an interval.

8. Test the function of $f(x) = x^{2020} + 2021x$ on $[2022, 2023]$ is Riemann integrable or not.

9. Under what circumstances differentiation and Riemann integration are inverse to each other.

10. Evaluate $lim(\frac{sinnx}{1+nx})$ for $x\epsilon R, x \geq 0$.

11. Show that the sequence of functions f_n defined on R as $f_n(x) = \frac{sin(nx+n)}{n}$ converges uniformly in R.

12. Do the limit of a convergent sequence of differentiable functions on an interval [a, b] is differentiable, if not what condition will make the limit function differentiable?

(10×2=20)

Part B

Answer any **six** *questions.*

Each question carries **5** *marks.*

13. Define Thomae's function on $(0, \infty)$ and show that it is continuous precisely at the irrational points in $(0, \infty)$.

14. State and prove Preservation of Intervals Theorem.

15. Let $I \subseteq R$ be an interval and let $f : I \to R$ be monotone on I. Then prove that the set of points $D \subseteq I$ at which f is discontinuous is a countable set .

16. Let $f : R \to R$ defined by $f(x) = \begin{cases} x^2, & x \: is \: rational \\ x, & x \: is \: irrational \end{cases}$, Prove that f is differentiable at $x = 0$.

17. Derive the inequality $x^{\alpha} \leq \alpha x + (1 - \alpha), \forall x \geq 0, 0 < \alpha < 1$

18. Evaluate the limit $\lim_{x \to \infty} x^{\frac{1}{x}}, x \in (0, \infty)$

19. Evaluate $\int_1^4 \frac{sin\sqrt{t}}{\sqrt{t}} dt.$

20. Evaluate $\int_0^2 t^2(1 + t^3)^{\frac{-1}{2}} dt.$

21. Suppose that (f_n) is a sequence of continuous functions on an interval I that converges uniformly on I to a function f. If $(x_n) \subseteq I$ converges to $x_0 \in I$, show that $lim(f_n(x_n)) = f(x_0)$.

(6×5=30)

Part C

Answer any **two** *questions.*

Each question carries **15** *marks.*

22. (a) Show that a function f is uniformly continuous on the interval (a,b) if and only if it can be defined at the endpoints a and b such that the extended function is continuous on [a,b].

(b) State and prove the Continuous Inverse Theorem.

23. (a) State and Prove L'Hospital's Rule I

(b) Using this, find the following

(i) $\lim_{x\to 0+} \dfrac{\tan x - x}{x^3}, x \in (0, \frac{\pi}{2})$

(ii) $\lim_{x\to 0+} \dfrac{\log \cos x}{x}$

24. (a) Let $f \in \mathcal{R}[a,b]$ and if $(\mathcal{P}_n)$ is any sequence of tagged partitions of $[a,b]$ such that $||\mathcal{P}_n|| \to 0$, prove that $\int_a^b f = lim_n S(f;\mathcal{P})$.

    (b) Suppose that f is bounded on $[a,b]$ and that there exists two sequences of tagged partitions $(\mathcal{P}_n)$ and $(\mathcal{Q}_n)$ of $[a,b]$ such that $||\mathcal{P}_n|| \to 0$ and $||\mathcal{Q}_n|| \to 0$, but such that $lim_n S(f;\mathcal{P}_n) \neq lim_n S(f;\mathcal{Q}_n)$. Show that $f \notin \mathcal{R}[a,b]$.

25. (a) State and prove the Cauchy Criterion for Riemann integrability of a function $f : [a,b] \to \mathbb{R}$.

    (b) Check the Riemann integrability of Dirichlet function.

(2×15=30)