QP CODE: 24020945

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Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) REGULAR EXAMINATIONS, APRIL 2024

Fourth Semester

Core Course - MM4CRT01 - VECTOR CALCULUS, THEORY OF NUMBERS AND LAPLACE TRANSFORMS

(Common for B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science)

2017 Admission Onwards

C8994E19

Time: 3 Hours

Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Give a vector equation for the line L through $P_0(x_0, y_0, z_0)$ parallel to $\mathbf{v}$.
2. Write the vector equation and component equation for a plane through $P_0(x_0, y_0, z_0)$ normal to $\mathbf{n} = A\mathbf{i} + B\mathbf{j} + C\mathbf{k}$.
3. Define the circle of curvature and centre of curvature for plane curves.
4. Find the accelaration for the position vector $r(t) = (2cost)i + (2sint)j$ at $t = 0$.
5. Find the potential function f for the field $F = 2xi + 3yj + 4zk$.
6. Find the divergence of the vector field $F = -yi + xj$.
7. Prove: If $a \equiv b \pmod{n}$ and $m|n$, then $a \equiv b \pmod{m}$.
8. Verify that $5^{38} \equiv 4 \pmod{11}$ using Fermat's theorem.
9. Show that $18! \equiv -1 \pmod{437}$.
10. Define Laplace transform of a function and hence prove that $\mathscr{L}(e^{at}) = \frac{1}{s-a}$.
11. State first shifting theorem for Laplace Transform.
12. Evaluate $\mathscr{L}(\sin^2 \omega t)$.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Graph the vector function $\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} + t\mathbf{k}$ giving the complete details.

14. The cylinder $f(x, y, z) = x^2 + y^2 - 2 = 0$ and the plane $g(x, y, z) = x + z - 4 = 0$ meet in an ellipse E. Find parametric equations for the line tangent to E at the point $P_0(1, 1, 3)$.

15. a) Explain component test for conservative field.
b) Give example of a field which is not conservative.

16. Using spherical co-ordinate system, find the surface area of a sphere of radius a.

17. Find the divergence and curl of $F = (xyz)i + (3x^2y)j + (xz^2 - y^2z)k$ at $(1, 2, -1)$.

18. Let a and b are integers that are not divisible by the prime p, then if $a^p \equiv b^p \pmod{p}$ prove that $a^p \equiv b^p \pmod{p^2}$.

19. Let n be a composite square-free integer, say, $n = p_1p_2 \cdots p_r$, where the p_i are distinct primes.
If $p_i - 1 | (n-1)$ for $i = 1, 2, \ldots, r$, then prove that n is an absolute pseudoprime.

20. Find $\mathscr{L}^{-1}\left\{\frac{s^2+2s+5}{(s-1)(s-2)(s-3)}\right\}$.

21. Using convolution theorem, solve $y'' + 5y' + 4y = 2e^{-2t},\ y(0) = 0,\ y'(0) = 0$.

(6×5=30)

Part C

Answer any ***two*** *questions.*

Each question carries ***15*** *marks.*

22.
1. Define the gradient vector of a function in the plane. Find an equation for the tangent to the curve $x^2 + y^2 = 4$ at the point $(\sqrt{2}, \sqrt{2})$.
2. Find the derivative of $f(x, y, z) = x^3 - xy^2 - z$ at $P_0(1, 1, 0)$ in the direction of $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$. In what direction does f change most rapidly at P_0, and what are the rates of change in these directions?

23. State and verify Green's Theorem (any one form) for the vector field $F(x, y) = (y - x)i + yj$ and the region bounded by the unit circle $C : r(t) = (sint)i + (cost)j, 0 \leq t \leq 2\pi$.

24.
1. Prove that the Euler phi-function is a multiplicative function.
2. Prove: For $n > 2, \phi(n)$ is an even integer.

25.
1. Solve $y'' + y' + 9y = 0,\ y(0) = 0.16,\ y'(0) = 0$ using Laplace Transform.
2. Solve the Volterra integral equation
$y(t) - \int_0^t (1 + \tau)y(t - \tau)d\tau = 1 - \sinh t$.