QP CODE: 24018064

24018064

Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, APRIL 2024

Fourth Semester

Elective - ME800403 - COMBINATORICS

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

684C72B4

Time: 3 Hours

Weightage: 30

*Instructions: (**Applicable for Private Registration, 2020 Admission Onwards**) This question paper contains two sections. Answer section I questions in the answer book provided. Section II Internal examination questions must be answered in the question paper itself. Follow the detailed instructions given under section II.*

SECTION I

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Define Multiplication Principle with an example
2. Find the number of ternary sequence of length 10 having two 0's,three 1's and five 2's
3. A) Find the number of integer solution to the equation $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 60$ where $x_1 \geq 2, x_2 \geq 5, 2 \leq x_3 \leq 7, x_4 \geq 1, x_5 \geq 3, x_6 \geq 2$
   B)Obtain the relation $S(r,n) = S(r-1,n-1) + nS(r-1,n) with\ r \geq n$ for Stirlings number of second kind
   C)Define Bell number
4. Construct a coloring of 5 clique using blue or red color which is independent of a blue 3-clique or a red 3-clique
5. Stating necessary results show that R(3,6)≤19
6. Define the function $\omega(m)$ in GPIE. Is $\omega(0)$=E(0).Justify?
7. Define F(n,m) and S(n,m)? Also find a relation connecting them
8. In the usual notations, define $D(n,r,k) and D_n$
9. Let a_r be the number of partitions of r into distinct parts of sizes 1,2,3 or 4. Then find the generating function for (a_r)
10. What is mean by r^{th} order linear homogeneous recurrence relation for a sequence (a_n)? Give example.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. *Find the number of ways of arranging the 26 letters in the English alphabets in a row such that there are exactly 5 letters between x and y.*

12. Let $X = \{1, 2, \ldots n\}$ and $Y = \{A \subset X \,|n \notin A\}$ and $Z = \{A \subset X \,|n \in A\}$.Using bijection principle Show that $|Y| = |Z|$

13. Among any group of 3000 people, Prove that there must be at least 9 who have the same birthday

14. Show that R(4,3)=9

15. State GPIE.Let $A_1, A_2, \ldots, A_n$ be n subsets of a finite set S. Give a formula for $|\bar{A}_1 \bigcap \bar{A}_2 \bigcap \ldots \bigcap \bar{A}_n|$ using GPIE

16. Find the number of non negative integer solutions using GPIE for the equation $x_1 + x_2 + x_3 = 11$ where $x_1 \leq 3, x_2 \leq 4, x_3 \leq 6$

17. A)What you mean by generating fnnction for a sequence (a_r)? Find the generating function for the sequence $(1, 2, 3, \ldots)$.
    B)Find the closed form of the generating functions for the sequence $(3r + 7)$ $where$ $r \in N^*$

18. Expalin the problem " Tower of Hanoi"

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. A) Let n and k be positive integers and let S be a set of n points in the plane such that no 3 points of S are collinear and for any point P of S, there are at least k points of S equidistant from P. Prove that $k < \frac{1}{2} + \sqrt{2n}$.
    B) Give the algebraic and combinatorial proof for $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$, where $n, r \in N$, with $r \leq n$.

20. A)Show that any set A of 13 distincts real nubers there are two points x and y in A such that $o < \frac{x-y}{1+xy} < 2 - \sqrt{3}$
    B)Let A be a set of m positive integers where $m \geq 1$.Show that there exist a nonempty prober subset B of A such that the sum $\sum(x|x \in B)$ is divisible by m

21. A)State and prove Principle of Inclusion and Exclusion for n finite sets
    B)Let A ,B,and C be finte sets Prove that $|\bar{A} \bigcap B| = |B| - |A \bigcap B|$ and $|\bar{A} \bigcap \bar{B} \bigcap C| = |C| - |A \bigcap C| - |B \bigcap C| + |A \bigcap B \bigcap C|$
    C)Find the number of integers in the set $\{1, 2, \ldots., 500\}$ which are not divisible by 5 nor by 7 but divisible by 3

22. (a) For each $r \in N^*$, find a_r, the number of ways of distributing r distinct objects into n distinct boxes such that no box is empty.
    (b) In how many ways can 4 of the letters from PAPAYA be arranged.

(2×5=10 weightage)