QP CODE: 24009036

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Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) SPECIAL REAPPEARANCE EXAMINATIONS, MARCH 2024

Fifth Semester

CORE COURSE - MM5CRT01 - MATHEMATICAL ANALYSIS

Common for B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science & B.Sc Computer Applications Model III Triple Main

2021 Admission Only

FCFCA3D9

Time: 3 Hours

Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Prove that the union of two disjoint denumerable sets are denumerable?
2. Prove that $a \times b = 0$ implies either$a = 0$ or $b = 0$
3. Define absolute value function?
4. Let $I_n = \left(0, \frac{1}{n}\right)$, $n \in N$ Prove that $\cap_{n=1}^{\infty} I_n = \phi$
5. Define convergent an ddivergent sequences. Give examples.
6. If a > 0, prove that $lim(\frac{1}{1+na}) = 0.$
7. Prove that (n) is divergent.
8. Prove that $(1+(-1)^n)$ is not Cauchy.
9. Let (x_n) and (y_n) be two sequences of real numbers and suppose that $x_n \leq y_n$ for all n. Prove that if lim $x_n = +\infty$ then lim $y_n = +\infty$.
10. Show that the harmonic series $\Sigma \frac{1}{n}$ diverges.
11. Is the series $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$ is absolutely convergent or not? Why?
12. Let $f : A \to \mathscr{R}$,and let $c \in \mathscr{R}$,then define the boundedness of a function f on a neighborhood of c.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. State and prove any two alternate definitions for supremum of a set?

14. Prove that If A, B are bounded sets then $Sup\ (A+B) = Sup\ A + Sup\ B$ where $A+B = \{a+b : a \in A, b \in B\}$

15. Let X = (x_n) and Y = (y_n) be sequences of real numbers that converges to x and y respectively and c ϵ **R**. Prove that the sequences cX converges to cx.

16. What is Euler number. Prove that Euler number lies between 2 and 3.

17. State and prove Monotone Subsequence Theorem.

18. State and prove the root test for the absolute convergence of a series in R.

19. State and prove Abel's Lemma.

20. If $f : A \to \mathscr{R}$ and if c is a cluster point of A, then prove that f can have only one limit at c.

21. Evaluate the one-sided limits of the function $h(x) = \frac{1}{(e^{\frac{1}{x}}+1)}$ at $x = 0$.

(6×5=30)

Part C

Answer any ***two*** *questions.*

Each question carries ***15*** *marks.*

22. (a.) State and Prove Nested interval property?
    (b.) Prove that the set of real numbers is not countable?

23. (a) State and prove Monotone Convergence Theorem.
    (b) Prove that (x_n) is divergent, where $x_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}$ forevery n ϵ**N**.

24. Test the convergence and absolute convergence of the following series.

    - $\sum_1^\infty \frac{(-1)^{n+1}}{(n^2+1)}$
    - Whose nth term is $\frac{n^n}{(n+1)^{n+1}}$

25. (a) Let $A \subseteq \mathscr{R}$, $f, g : A \to \mathscr{R}$, and let $c \in \mathscr{R}$ be a cluster point of A, Suppose that $f(x) \leq g(x)$ for all $x \in A$, $x \neq c$, Then prove the following

    - If $\lim_{x \to c} f = \infty$, then $\lim_{x \to c} g = \infty$.
    - If $\lim_{x \to c} g = -\infty$,then $\lim_{x \to c} f = -\infty$.

    (b) Give an example of a function that has a right-hand limit but not a left-hand limit at a point.
    (c) . Evaluate the limit or show that it do not exist " $\lim_{x \to 1} \frac{x}{x-1}$ where $x \neq 1$.

(2×15=30)