QP CODE: 24018745

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Reg No : ....

Name : ....

MSc DEGREE (CSS) EXAMINATION , APRIL 2024

Second Semester

CORE - ME010204 - COMPLEX ANALYSIS

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 Admission Onwards

6B32491C

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Differentiate between point wise convergence and uniform convergence of sequence of functions.
2. Find the symmetric points of i and 1 w.r.t the circle $|z-3|=1$.
3. Prove fundamental inequality for an arbitrary complex function f(t).
4. If the closed curve γ lies inside of a circle , then prove that the index n(γ,a) is zero for all points outside the circle.
5. Evaluate $\int_{|z|=2}\frac{dz}{z^2+1}$.
6. State the Cauchy's integral formula for higher derivatives. Evaluate $\int\limits_{|z|=2}\frac{e^{2z}}{(z+1)^4}dz.$
7. Suppose that f(z) is analytic in a region Ω' obtained by omitting a point a from Ω . State a necessary and sufficient condition for $f(z)$ to have an analytic extension in Ω.
8. Prove that the poles of a meromorphic function cannot have a limit point.
9. What do you mean by 'a cycle in a region is homologous to zero'?
10. Define connectivity of a multiply connected region.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Find the correspondence between the coordinates of a point on the Riemann sphere and its image in the complex plane.

12. Show that an analytic function f(z) in a region W reduces to a constant if its modulus or argument is a constant.

13. Define rectifiable arcs. State and prove the necessary and sufficient condition for an arc to be rectifiable.

14. If f(z) is an analytic function in an open disk Δ, then evaluate $\int_{\gamma} f(z)dz$ where γ is any closed curve in Δ.

15. Show that an isolated singularity of $f(z)$ cannot be a pole of $e^{f(z)}$.

16. State and prove the theorem on local correspondence.

17. Write a brief note on calculus of residues.

18. How many roots of the equation $z^4 - 6z + 3 = 0$ have their modulus between 1 and 2?

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. (i)Prove that the set of all linear transformations form a group under composition mapping.

(ii)Show that a linear transformation $w = \frac{az+b}{cz+d}$, $c \neq 0$ is composed by a translation, an inversion, a rotation and a homothetic transformation.

(iii) Show that cross ratio is invariant under a linear transformation.

20. State and prove Cauchy's stronger theorem for rectangles.

21. (a)State and prove the maximum principle.

(b) Let $f(z)$ be analytic in a region Ω and $a \in \Omega$ such that $|f(a)| \leq |f(z)|$ for every $z \in \Omega$ then prove that either $f(a) = 0$ or $f(z)$ is a constant.

22. Evaluate $\int_0^{\infty} \frac{x^{2m} dx}{x^{2n}+1}$; $n > m$.

(2×5=10 weightage)