QP CODE: 24019463

24019463

Reg No : .....

Name : .....

B.Sc DEGREE (CBCS) REGULAR / IMPROVEMENT / REAPPEARANCE EXAMINATIONS, MAY 2024

Second Semester

Core Course - MM2CRT01 - MATHEMATICS - ANALYTIC GEOMETRY, TRIGONOMETRY AND DIFFERENTIAL CALCULUS

(Common for B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science)

2017 ADMISSION ONWARDS

0835921B

Time: 3 Hours Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Derive the condition that the line y = mx + c is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.
2. Show that two tangents can be drawn from any point to a hyperbola.
3. Find the orthoptic locus of the parabola $y^2 = 4ax$.
4. Derive the equation of chord of contact of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.
5. Find the cartesian coordinate corresponding to the polar coordinate (3,0).
6. Find the equation for a circle centered at the pole. Give an example.
7. Prove that sin(x+y) = sinx cosy + cosx siny.
8. Define Hyperbolic functions.
9. Factorize $x^9 + 1$.
10. Find the n^{th} derivative of $log_e(ax + b)$.
11. Using Leibnitz's theorem find the n^{th} derivative of $e^x logx$.
12. State L' Hospital's rule of limits.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Prove that the tangent at the exteremity of a diameter of a parabola is parallel to the system of chords it bisects.

14. The polar of a point P with respect to the parabola $y^2 = 4ax$ meets the curve in Q and R. Show that if P lies on the line $lx + my + n = 0$, then the middle point of QR lies in the parabola $l(y^2 - 4ax) + 2a(lx + my + n) = 0$.

15. Show that the locus of the poles of normal chords of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is the curve $y^2a^6 - x^2b^6 = (a^2+b^2)^2x^2y^2$.

16. Prove that the acute angle between two conjugate diameters of an ellipse is minimum when they are equal. Also find the minimum angle.

17. If the normal at P on a conic meets the axis in G, then $SG = e.SP$, where S is the focus.

18. If $\sin(A + iB) = x + iy$, show that

    (i) $\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1$

    (ii) $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$

19. Sum the series $1 + c\cosh\alpha + c^2\cosh 2\alpha + \ldots + c^{n-1}\cosh(n-1)\alpha$., where c is less than unity.

20. If $y = [x + \sqrt{1+x^2}]^m$, show that $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - m^2y = 0$

21. Find $lim_{x\to 0}[\frac{tanx}{x}]^{\frac{1}{x}}$.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. Prove that

    (a) The tangents at the extremities of a diameter of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, are parallel to the diameter conjugate to it.

    (b) The tangents at the extremities of a chord of an ellipse will intersect on the diameter conjugate to the diameter parallel to the given chord.

    (c) The eccentric angles of the ends of a pair of conjugate diameters differ by a right angle.

23. If two conics have a common focus, show that two of their common chords pass through the point of intersection of their directrices.

24. Sum the series

(i) $ccos\alpha - \frac{1}{3}c^3 cos(\alpha + 2\beta) + \frac{1}{5}c^5 cos(\alpha + 4\beta) - \ldots.$

(ii) $sin\alpha sin\beta + \frac{1}{2} sin2\alpha sin2\beta + \frac{1}{3} sin3\alpha sin3\beta + \ldots$

25. (a) Show that the nth derivative of $y = \frac{1}{1+x+x^2+x^3}$ is $\frac{1}{2}(-1)^n n! sin^{n+1}\theta [sin(n+1)\theta - cos(n+1)\theta + (sin\theta + cos\theta)^{-n-1}]$ where $\theta = cot^{-1}x$.

(b) Find the nth derivative of $\frac{1}{(x-1)^2(x-2)}$.

(2×15=30)