QP CODE: 24027207

24027207

Reg No : .....

Name : .....

B.Sc DEGREE (CBCS) REGULAR / IMPROVEMENT / REAPPEARANCE EXAMINATIONS, OCTOBER 2024

Third Semester

Core Course - MM3CRT01 - CALCULUS

Common to B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

467CD5C0

Time: 3 Hours

Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Using Taylor's series , expand $f(x) = \frac{1}{x-1}$ in powers of (x-2).
2. Find the points of inflection of the curve $y = 3x^4 - 4x^3 + 1$.
3. Find the centre of curvature of the given point on the curve $y = x^2$; (1/2, 1/4).
4. Define envelope of one parameter family of curves.
5. If $f(x, y) = x - y$, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$
6. Find $\frac{dw}{dt}$ if $w = xy$, $x = \cos t$, $y = \sin t$.
7. Explain the absolute minimum of a continuous function at a point (a, b) defined on a bounded region R.
8. Find the volume of the solid of cross sectional area $A(x) = 2x + 1$ from $x = 1$ to $x = 2$.
9. Obtain the volume of solid of revolution generated by rotating the region between the Y-axis and graph of the function $x = g(y)$; $c \leq y \leq d$ about Y-axis.
10. If $R(y)$ and $r(y)$ denote the outer and inner radius of cross section of a solid of revolution about Y-axis, with hole at y ; $c \leq x \leq d$. Find the volume of solid.

11. Write an equivalent double integral of $\int_0^1 \int_2^{4-2x} dy\,dx$ with the order of integration reversed.

12. Write the formula for finding average value of a function $f(x, y)$ over a region R in XY-plane..

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Obtain the expansion of $\log\cosh x$ in powers of x by Maclaurin,s series.

14. Find the asymototes parallel to the co-ordinates axes of the curve $(x^2 + y^2)x - ay^2 = 0$.

15. Verify that $w_{xy} = w_{yx}$ where $w = x^2 \tan(xy)$.

16. Find all local extreme values and saddle point, if any, of the function $f(x, y) = x^3 - y^3 - 2xy + 6$.

17. Find the volume of the solid generated by revolving the region bounded by the curves and lines $y = x^2$, $y = 2 - x$, $x = 0$ for x \geq 0 about the Y-axis using shell method.

18. Find the length of the curve $y = \int_0^x \tan t\,dt$, $0 \le x \le \pi/6$

19. Find the average value $f(x, y, z) = x + y + z$ over the cubical region D bounded by the coordinate planes $x = 2, y = 2$ and $z = 2$ in the first octant.

20. Evaluate the spherical integral $\int_0^{\pi} \int_0^{\pi/4} \int_0^2 (\rho\cos\phi)\,\rho^2 \sin\phi\,d\rho\,d\phi\,d\theta$.

21. Find the image under the transformation $u = 3x + 2y$, $v = x + 4y$ of the triangular region in the XY-plane bounded by the X-axis , Y-axis and the line $x + y = 1$. Sketch the transformed region in the UV-plane.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. a) Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

b) Find the envelope of the line $\frac{x}{a} + \frac{y}{b} = 1$ where the parameters a and b are connected by the relation $a^2 + b^2 = c^2$.

23. (a). If $\sin u = \dfrac{x+y}{\sqrt{x}+\sqrt{y}}$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \dfrac{1}{2}\tan u$.

(b). Find the maximum and minimum values that the function $f(x, y) = 3x + 4y$ takes on the circle $x^2 + y^2 = 1$

24. (a). Using the shell method to find the volumes of the solids generated by revolving the regions bounded by the lines and curves

$y = x + 2$; $y = x^2$ about (i) the X-axis (ii) the line $x = 2$.

(b). Find the area of the surface generated by revolving the curve

$y = 2\sqrt{x}$; $1 \leq x \leq 2$ about the X-axis.

25. (a). Evaluate $\displaystyle\int_0^1 \int_0^{1-x^2} \int_3^{(4-x^2-y)} x \, dz \, dy \, dx$

(b). Evaluate the cylindrical coordinate integral $\displaystyle\int_0^{2\pi} \int_0^1 \int_r^{\sqrt{2-r^2}} dz \, r \, dr \, d\theta$

(2×15=30)