QP CODE: 24035580

24035580

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS, OCTOBER 2024

Fifth Semester

CORE COURSE - MM5CRT03 - ABSTRACT ALGEBRA

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

186FC449

Time: 3 Hours

Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Check whether the usual multiplication is a binary operation on the set $\mathbb{R}^+$.
2. State whether True or False:
   a) "Each element of a group appear once and only once in each row and column of the group table".
   b) "There is only one group of three elements , upto isomorphism".
3. Define a cyclic group.
4. Find the number of elements in the set $\{\sigma \in S_4 | \sigma(3) = 3\}$.
5. Find all orbits of the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 2 & 4 & 8 & 3 & 1 & 7 \end{pmatrix}$.
6. Find the index of $< 3 >$ in the group $\mathbb{Z}_{24}$.
7. Define the direct product of the groups $G_1, G_2, \cdots, G_n$.
8. Let G be a group and H be normal subgroup of G. Find the identity element in the factor group G/H.
9. Show that S_n is not a simple group when n$\geq$ 3.
10. Show that the matrix $\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$ is a divisor of zero in M2(Z).
11. Prove that Zp is a field if p is a prime.
12. Mark each of the following true or false.
    a) A ring homomorphism $\phi : R \to R'$ carries ideals of R into ideals of R'
    b) A ring homomorphism is one to one if and only if the kernel is { 0 }.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Check whether $\langle\mathbb{C},\cdot\rangle$ and $\langle\mathbb{R},\cdot\rangle$ under usual multiplication are isomorphic.

14. Define subgroup of a group. Give two examples.

15. Find the quotient q and remainder r when -50 is divided by 8 according to the division algorithm.

16. Prove from linear algebra that no permutation in S_n can be expressed both as a product of an even number of transpositions and as a product of an odd number of transpositions.

17. Prove that for $n \geq 2$, the number of even permutations in S_n is the same as the number of odd permutations. Define the alternating group A_n on n letters.

18. Show that composition of group homomorphisms is again a group homomorphism.

19. Let G be a group. Show that $Inn(G)$ the set of all inner automorphisms of G is anormal subgroup of $Aut(G)$, the group of all automorphisms of G.

20. Prove that 1) 0.a = a.0 = 0 2) a (-b) = (-a) b = -(ab) 3) (-a) (-b) = ab where R is a ring with additive identity 0, and $a, b \in R$.

21. Let N be an ideal of a ring R . Prove that $\gamma : R \to R/N$ given by $\gamma(x) = x + N$ is a ring homomorphism with kernel N.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. a) Define an abelian group.

    b) Show that the subset S of $M_n\ (R\,)$ consisting of all invertible $n \times n$ matrices under matrix multiplication is a group.

    Also check whether it is an abelian group.

    c) Prove that $\langle\mathbb{Q}^{+}, *\rangle$ is a group where, $*$ is defined by $a * b = ab/2$.

23.
    1. Let H be a subgroup of a group G. Let the relation $\sim_R$ be defined on G by $a \sim_R b$ if and only if $ab^{-1} \in H$. Then show that $\sim_R$ is an equivalence relation on G. What is the cell in the corresponding partition of G containing $a \in G$?
    2. Let H be a subgroup of a group G. Then define the left and right cosets of H containing $a \in G$.

3. Let H be the subgroup $< \mu_1 >= \{\rho_0, \mu_1\}$ of S_3. Find the partitions of S_3 into left cosets of H.

24. State and prove fundamental homomorphism theorem.

25. a) Let p be a prime . Show that in a ring Zp , (a + b)p = ap + bp for all $a, b \in Z_p$

b) Show that if a and b are nilpotent elements of a commutative ring , then a + b is also nilpotent.

c) Show that intersection of subrings of a ring R is again a subring of R.

(2×15=30)