QP CODE: 24044651

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Reg No : ....

Name : ....

M.Sc DEGREE (CSS) EXAMINATION, OCTOBER 2024

Third Semester

M.Sc MATHEMATICS , M.Sc MATHEMATICS (SF)

CORE - ME010301 - ADVANCED COMPLEX ANALYSIS

2019 ADMISSION ONWARDS

AF6C7C78

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Define a symmetric region and give an example.
2. What do you mean by the mean value propery of a real valued function $u(z)$ in a region Ω?
3. Write the Taylor's series expansions of e^z and sinz about the origin.
4. Define Canonical product.
5. Prove that $\Gamma(z)\Gamma(1-z)=\frac{\pi}{sin\pi z}$.
6. *Define Riemann zeta function. Also check whether the number of primes is finite or not.*
7. *Using Riemann functional equation prove* $\zeta(1-s)=2^{1-s}\pi^{-s}cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)$.
8. *Where do the zeros of zeta function lie in the complex plane?*
9. What is meant by the boundary behavior?
10. Prove that $\frac{\sigma'(z)}{\sigma(z)}=\zeta(z)$.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. State and prove maximum principle for harmonic functions.
12. State and prove Schwarz theorem.

13. Write the general form of a Laurent series for $f(z)$ which is analytic in the annulus $R_1 < |z-a| < R_2$. Derive the Laurent series of $f(z) = \frac{e^z}{(z+1)^2}$ about $z = -1$.

14. State Mittag-Leffler's theorem. Prove that $\pi cosec \pi z = \lim_{n \to \infty} \sum_{n=-m}^{m} (-1)^n \frac{1}{z-n}$.

15. *Define normal family and totally bounded family of functions. Prove that a sequence of functions converges uniformly to f on compact subsets if and only if it converges to f with respect to ρ.*

16. *If a family $\mathcal{F}$ of continuous functions with values in a metric space S is normal in a region Ω of the complex plane then prove that $\mathcal{F}$ is equicontinuous on every compact subsets of Ω.*

17. Prove that a non constant elliptic function has equally many zeros and poles.

18. Prove that $\zeta(z+u) = \zeta(z) + \zeta(u) + \frac{1}{2} \frac{\wp'(z) - \wp'(u)}{\wp(z) - \wp(u)}$.

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any ***two*** *questions.*

Weight ***5*** *each.*

19. Define a subharmonic function. State and prove any three properties.

20. Obtain Jensen's formula. Deduce Poisson-Jensen formula.

21. (i) Describe the Riemann Zeta function.
(ii) Prove that the Zeta function can be extended to the whole plane as a meromorphic function having a single, simple pole at $s = 1$.
(iii) Find the residue of the Zeta function at $s = 1$.

22. (a) Prove that any simply connected region other than the complex plane is topologically equivalent to the unit disk.
(b) Prove that the Riemann mapping is unique.

(2×5=10 weightage)