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M.Sc DEGREE (CSS) EXAMINATION, OCTOBER 2024

Third Semester

M.Sc MATHEMATICS , M.Sc MATHEMATICS (SF)

CORE - ME010302 - PARTIAL DIFFERENTIAL EQUATIONS

2019 ADMISSION ONWARDS

6BDA4900

Time: 3 Hours   Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. *Find the integral curves of* $\frac{dx}{x^2(y^3-z^3)} = \frac{dy}{y^2(z^3-x^3)} = \frac{dz}{z^2(x^3-y^3)}$
2. *Verify that the equation* $yz(y+z)\,dx + xz(x+z)\,dy + xy(x+y)\,dz = 0$ *is integrable.*
3. What is the general form of the linear partial differential equation in n variables. Explain how a general solution of this equation is found.
4. Verify that the equation $z = \sqrt{(2x+a)} + \sqrt{2y+b}$ is a complete integral of the partial differential equation $z = \frac{1}{p} + \frac{1}{q}$.
5. What is meant by compatible systems of first order equations? State the condition under which the partial differential equations $f(x,y,z,p,q) = 0$ and $g(x,y,z,p,q) = 0$ are compatible?
6. Find the complementary function of $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$.
7. Prove $F(D,D')e^{ax+by} = F(a,b)e^{ax+by}$.
8. Find the particular integral of $[D^2 - D']z = 2y - x^2$.
9. Establish a formula for finding the potential function of a family of equipotential surfaces.
10. Show that the real and imaginary parts of an analytic function are harmonic

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. *Find the equations of the system of curves on the cylinder* $2y = x^2$ *orthogonal to its intersections with the hyperboloids of the one-parameter system* $xy = z + c$

12. Eliminate the arbitrary function f from the given equations.
a) $f(x^2+y^2+z^2,\ z^2-2xy)=0$
b) $z=xy+f(x^2+y^2)$

13. Find a complete integral of the equation $p^2x+q^2y=z$.

14. Show that the differential equation $2xz+q^2=x(xp+yq)$ has a complete integral $z+a^2x=axy+bx^2$ and deduce that $x(y+hx)^2=4(z-kx^2)$ is also a complete integral.

15. By Jacobi's method, solve $z^2=pqxy$.

16. Verify that the PDE $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial y^2}=\frac{2z}{x^2}$ is satisfied by $z=\frac{1}{x}\phi(y-x)+\phi'(y-x)$.

17. By separating the variables show that the equation $\nabla_1^2 V=0$ has solutions of the form $Aexp(nx\pm iny)$ where A and n are constants.Deduce that the functions of the form $V(x,y)=\sum_r A_r e^{\frac{-r\pi x}{a}} sin\frac{r\pi y}{a}$, $x\geq 0, 0\leq y\leq a$ where $A_r's$ and $B_r's$ are constants, are the plane harmonic functions satisfying the conditions $V(x,y)=0, V(x,a)=0, V(x,y)\to 0$ as $x\to\infty$

18. Show that in cylindrical coordinates ρ, z, ϕ , the Laplace's equation has solutions of the form $R(\rho)exp(\pm mz\pm in\phi)$ where $R(\rho)$ is a solution of Bessel's equation
$\frac{d^2R}{d\rho^2}+\frac{1}{\rho}\frac{dR}{d\rho}+(m^2-\frac{n^2}{\rho^2})R=0.$

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. Prove the following.
a) A Pfaffian differential equation in two variables always possesses an integrating factor.
b) A necessary and sufficient condition that there exists between two functions $u(x,y)$ and $v(x,y)$ a relation $F(u,v)=0$ not involving x or y explicitly is that $\frac{\partial(u,v)}{\partial(x,y)}=0.$

20. $a)$ Find the general solution of the equation $2x(y+z^2)p+y(2y+z^2)q=z^3$ and deduce that $yz(z^2+yz+-2y)=x^2$ is a solution.
$b)$ Find the general integral of the equation $(x-y)p+(y-x-z)q=z$ and the particular solution through the circle $z=1, x^2+y^2=1.$

21. Reduce the equation to canonical form and solve $u_{xx}+2u_{xy}+u_{yy}=0.$

22. (a) State and prove the necessary condition that a family of surfaces $f(x,y,z)=c$ is a family of equipotential surfaces.
(b) Show that the surfaces $(x^2+y^2)^2-2a^2(x^2-y^2)+a^4=c$ can form a family of equipotential surfaces and find the general form of the corresponding potential function.

(2×5=10 weightage)