QP CODE: 24044655

24044655

Reg No : .....................

Name : .....................

# M.Sc DEGREE (CSS) EXAMINATION, OCTOBER 2024

## Third Semester

M.Sc MATHEMATICS , M.Sc MATHEMATICS (SF)

## CORE - ME010303 - MULTIVARIATE CALCULUS AND INTEGRAL TRANSFORMS

2019 ADMISSION ONWARDS

CB7D0E57

Time: 3 Hours

Weightage: 30

### Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Find the Fourier Series for $f(x) = 4x, 0 < x < 2\pi$
2. Show by an example that Lebesgue integrability of $f$ and $g$ alone will not give a convolution integral of $f$ and $g$.
3. Define total derivative of a function $\mathbf{f} : S \to \mathbf{R}^m$, $S \subseteq \mathbf{R}^n$. Show that if $\mathbf{f}$ is differentiable at $\mathbf{c}$, then $\mathbf{f}$ is continuous at $\mathbf{c}$.
4. Give matrix representation for a linear function $T : \mathbf{R}^n \to \mathbf{R}^m$.
5. Let $\mathbf{f} : S \to \mathbf{R}^m$ be differentiable at each point of an open connected subset $S$ of $\mathbf{R}^n$. Show that if $\mathbf{f}'(\mathbf{c}) = 0; \forall c \in S$, then $\mathbf{f}$ is constant on $S$.
6. Define Jacobian determinant and find the Jacobian determinant for the function $f(z)3z^2 + z$
7. Define open mapping. State and prove a sufficient condition for a mapping to carry open sets onto open sets.
8. Define a Stationary point and a Saddle point.
9. *Let* $G(x) = \sum_{i \neq m} x_i e_i + g(x) e_m$, $x \in E$ *be a primitive mapping and* $(D_m g)(a) \neq 0$. *Prove that* $G'(a)$ *is invertible.*
10. Define k- form in $E \subseteq R^n$.

(8×1=8 weightage)

### Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. State and prove Weierstrass Approxiamation Theorem for real valued and continuous functions on compact interval.
12. If $p > 0, q > 0$, prove that the beta function can be expressed using gamma function as $\mathcal{B}(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$
13. a. Show that if $f(x) = \|x\|^2$ and $F(t) = f(\mathbf{c} + t\mathbf{u})$, then $F'(t) = 2\mathbf{c}.\mathbf{u} + 2t\|\mathbf{u}\|^2$.
    b. Calculate all partial derivatives and directional derivatives of $f(\mathbf{x}) = \mathbf{a}.\mathbf{x}; \mathbf{a} \in R^n$ defined on $\mathbf{R}^n$.
14. a. Define matrix of a linear function, $T : \mathbf{R}^n \to \mathbf{R}^m$.
    b. Define Jacobian matrix. Explain how Jacobian matrix is related to the gradient vector.
15. State inverse function theorem and implicit function theorem .
16. (a) Define stationary point and saddle point of a function from $R^n \to R$
    (b) Find the saddle point of the function $f(x,y) = x^2 - 4xy + y^2 + 6y + 2$

17. $Define\ support\ of\ a\ function\ f\ on\ R^k.\ Also\ show\ that\ for\ every\ f \in C(I^k), L(f) = L'(f).$

18. $(a) If\ \gamma(t) = (acost, bsint),\ 0 \leq t \leq 2\pi\ then\ find\ \int_\gamma xdy\ and\ \int_\gamma ydx.$

$(b) Let\ \gamma\ be\ a\ 1-surface\ in\ R^3\ with\ parameter\ domain\ [0,1]\ and\ \omega = xdy + ydx. Then\ prove\ that\ \int_\gamma \omega$ $depends\ only\ on\ the\ endpoint\ of\ the\ curve\ \gamma$

(6×2=12 weightage)

**Part C (Essay Type Questions)**

*Answer any **two** questions.*

*Weight **5** each.*

19. State and prove the Exponential form of Fourier Integral Theorem.

20. State and prove the chain rule.

21. Assume that one of the partial derivatives $D_1 f, D_2 f, \ldots . D_n f$ exist at c and the remaining $n-1$ partial derivatives exist in some n- ball $B(c)$ and are continuous at $c$. Prove that $f$ is differentiable at c

22. $Suppose\ F\ is\ a\ C'\ mapping\ of\ an\ open\ set\ E \subset R^n\ into\ R^n,\ 0 \in E,\ F(0) = 0\ and\ F'(0)\ is\ invertible.$ $Then\ there\ is\ a\ nbd\ of\ 0\ in R^n\ in\ which\ F(x) = B_1 B_2 \ldots . B_{n-1} G_n \circ \ldots . \circ_1 G(x).$

(2×5=10 weightage)