QP CODE: 24035574

24035574

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS, OCTOBER 2024

Fifth Semester

CORE COURSE - MM5CRT01 - MATHEMATICAL ANALYSIS

Common for B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science & B.Sc Computer Applications Model III Triple Main

2017 Admission Onwards

2D5A9EF5

Time: 3 Hours

Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Prove that the set of all integers Z is denumerable.
2. Prove that there doesnot exist any smallest positive real number.
3. Find all $x \in R$ such that $|x-1| > |x+1|$.
4. Is any intervals are finite set? Justify.
5. Define sequence of real numbers. What is Fibonacci sequence.
6. Show that $lim(\frac{1}{n} - \frac{1}{n+1}) = 0.$
7. If X = (x_n) is a convergent sequence of real numbers such that $x_n \geq 0$ for every n, then prove that x = $\lim(x_n) \geq 0$.
8. Let X = (2, 4, 6,..., 2n,...) and Y = $\left(1, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{n}, \ldots\right)$. Find X + Y and X.Y.
9. Let X = (x_n) be a bounded sequence of real numbers and let every convergent subsequence of X converge to x. Prove that sequence X also converges to x.
10. If a series in $\mathbb{R}$ is absolutely convergent, then it is convergent.
11. Test the convergence of $\sum_1^{\infty} \frac{(-1)^{n+1}}{(n^2+1)}$
12. Show that $\lim\limits_{x \to c} x^3 = c^3$ for any $c \in \mathscr{R}$.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Prove that If A, B are bounded sets then $Sup\ (A+B) = Sup\ A + Sup\ B$ where $A+B = \{a+b : a \in A, b \in B\}$.

14. Prove that $x \in [0,1]$ then the binary representation of x forms a sequnce consisting only $0, 1$.

15. What is Euler number. Prove that Euler number lies between 2 and 3.

16. State and prove Cauchy Convergence Criterion.

17. Let (x_n) and (y_n) be two sequences of real numbers and suppose that $x_n \leq y_n$ for all n. Prove that

(a) if $\lim x_n = +\infty$ then $\lim y_n = +\infty$.

(b) if $\lim y_n = -\infty$ then $\lim x_n = -\infty$.

18. State and prove the root test for the absolute convergence of a series in R.

19. If (x_n) is a monotone convergent sequence and Σy_n is convergent, then establish the convergence of $\Sigma x_n y_n$.

20. Check whether the one-sided limits of the function $g(x) = e^{\frac{1}{x}}$ at $x = 0$ exist or not.

21. Give an example of a function that has a right-hand limit but not a left-hand limit at a point.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. Prove that there exist a real number x such that $x^2 = 2$.

23. (a) State and prove Monotone Convergence Theorem.

(b) Let Y = (y_n) be the sequence defined as $y_1 = 1$ and $y_{n+1} = \frac{2y_n+3}{4}, n \geq 1$. Prove that $\lim Y = \frac{3}{2}$.

24. (a) State and prove the Limit Comparison Test for the convergence of series.

(b) Discuss the convergence of

- $\sum_1^\infty \frac{1}{n^2-n+1}$
- $\sum_1^\infty \frac{1}{\sqrt{n+1}}$

25. (a) Let $A \subseteq \mathscr{R}$, $f : A \to \mathscr{R}$ and let $c \in \mathscr{R}$ be a cluster point of A. If $a \leq f(x) \leq b$ for all $x \in A, x \neq c$, and if $\lim_{x \to c} f$ exists,Then prove that $a \leq \lim_{x \to c} f \leq b$.

(b) Check whether the following limits exist or not. Give explanations.

(1) $\lim_{x \to 0} sinx$ (2) $\lim_{x \to 0} (\frac{cosx-1}{x})$

(2×15=30)