QP CODE: 24044659

24044659

Reg No : .....

Name : .....

M.Sc DEGREE (CSS) EXAMINATION, OCTOBER 2024

Third Semester

M.Sc MATHEMATICS , M.Sc MATHEMATICS (SF)

CORE - ME010305 - OPTIMIZATION TECHNIQUE

2019 ADMISSION ONWARDS

00D1A61D

Time: 3 Hours Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Write short note about linear programming problem.
2. What do you mean by degeneracy in LP Problems?
3. *Solve graphically: Max* $5x_1 - 4x_2$ *subject to* $3x_1 + 5x_2 \leq 15, 4x_1 + 3x_2 \leq 12, x_1 \geq 0, x_2 \geq 0$.
4. *Define integer vector and hence define ILPP*
5. Define the following with suitable example.
   (i) Graph (ii) Cycle (iii) Arborescence
6. What you mean by minimum path problem?
7. Explain the term critical path.
8. If $f(x) = x^2 - 5x + 4$, (1) calculate the value of this function at x = 3 and (2) find the value of f(x) at x=6 by using (1) and derivatives of f(x).
9. What you mean by perturbation?
10. Write down the Lagrange function and K-T conditions of NLP
    Maximize $f(x) = 8x_1 + 10x_2 - x_1^2 - x_2^2$ subject to $3x_1 + 2x_2 \leq 6; x_1, x_2 \geq 0$.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Write the dual of the following LP problem and verify that the dual of the dual is primal.
    Maximize $f(X) = 2x_1 + 3x_2 + x_3$, subject to
    $4x_1 + 3x_2 + x_3 \geq 6, x_1 + 2x_2 + 5x_3 \leq 4$ *and* $x_1, x_2, x_3 \geq 0$.

12. (a) Write a short note on Applications of Duality.
    (b) When we apply Dual Simplex Method?
    (c) To find the optimum value of a primal problem, it is enough to find the optimum value of its dual. Isn't true? Justify.

13. *Develop the theory of Cutting plane to solve an ILPP.*

14. *Write the mathematical model of Knapsack problem and hence solve the Knapsack problem with Knapsack capacity W= 12*

Object	Weight	Value
1	2	10
2	2	14
3	3	18
4	6	48
5	8	80

15. Write short note about goal programming
    A factory can manufacture two products A and B. The profit on a unit of A is Rs. 80 and of B is Rs. 40. The maximum demand of A is 6 units per week and B is 8 units per week. This manufacturer has set a goal of achieving a profit of Rs. 640 per week. Formulate the problem as goal programming and solve it.

16. Prove that the maximum flow in a graph is equal to the minimum of the capacities of all possible cuts in it.

17. Express the function $2x_1^2 + 2x_2^2 + 4x_3^2 + 2x_1x_2 + 2x_1x_3 + 4x_2x_3$ in the form $X'QX$. Is it positive definite?

18. Minimize $f(X) = (x_1 - 2)^2 + (x_2 - 1)^2$ subject $x_1 - 2x_2 - 1 = 0$

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. Solve the following LPP using simplex method
    Minimize $f(X) = x_1 - 3x_2 + 2x_3$
    Subject to
    $3x_1 - x_2 + 3x_3 \leq 7, -2x_1 + 4x_2 \leq 12, -4x_1 + 3x_2 + 8x_3 \leq 10; x_1, x_2, x_3 \geq 0$

20. *Solve the ILPP using Branch and Bound method* $Min\ z = 5x_1 + 7x_2$ *subject to* $2x_1 + x_2 \leq 13, 5x_1 + 9x_2 \leq 41, x_1 \geq 0. x_2 \geq 0$ *and* x_1, x_2 *are integers.*

21. Five villages in a hilly region are to be connected by roads. The direct distance in km between each pair of villages along a possible road and the cost of construction per km(104m Rs) are given in the following table. Find minimum cost at which all the villages can be connected by roads which should constructed.

	Distance					
Cost		1	2	3	4	5
	1	---	18	12	15	10
	2	3	---	15	8	22
	3	4	3	---	6	20
	4	5	5	6	---	7
	5	2	2	5	7	---

22. Minimize the function $f(x) = 3x^4 + (x-1)^2, 0 \leq x \leq 4$ using Golden Section Search given the resolution parameter $\epsilon = 0.1$.

(2×5=10 weightage)