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Reg. No...................................

Name.......................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, JUNE 2019

Second Semester

Faculty of Science

Branch I (A)—Mathematics

MT 02 C 07—ADVANCED TOPOLOGY

(2012 Admissions)

Time : Three Hours | Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1 Define Topological coproduct. Give example to show that properties of coproducts are dual to those of products.

2 A product of topological spaces is completely regular iff each co-ordinate space is so—Prove.

3 Obtain a condition under which the evaluation function is one-to-one.

4 Define and explain : Evaluation function.

5 Define a directed set with two examples.

6 Prove : A space is Hausdorff iff every ultra filter converges to atmost one point in it.

7 Show that a first countable, countably compact space is sequentially compact.

8 Explain compactification of a topological space when it is called n-point compactification ?

(5 × 1 = 5)

Part B

Answer any **five** *questions.*
Each question has weight 2.

9 Prove that the projection functions are open.

10 Prove : For any sets Y, I and J, we have $\left(Y^{I}\right)^{J} = Y^{I \times J}$ upto a bijection.

11 Let H be a group and $\{f_i : H \to G_i / i \in I\}$ be indexed family of group homomorphisms. Prove that the evaluation function $e : H \to \underset{i \in I}{\pi} G_i$ is a group homomorphism.

Turn over

12 Show that every filter is contained in an ultrafilter.

13 Prove that a topological space is Hausdorff iff limits of all nets in it are unique.

14 Prove that in an indiscrete space, every net converges to every point and that this property characterises indiscrete spaces.

15 Prove that local compactness is preserved under continuous open functions and that it is a weakly hereditary property.

16 Let X be a Hausdorff space and Y be a dense subset of X. If Y is locally compact in the relative topology on it, prove that Y is open in X.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question has weight 5.

17 State Urysohn characterisation of normality-and Tietze characterisation of normality. Also prove that the latter implies the former.

18 Prove : A product of spaces is connected iff each co-ordinate space is connected. Also explain why the proof of this theorem is peculiar.

19 Establish the embedding lemma.

20 State and prove the relationship between :

(i) Cluster points and subnets.

(ii) Prove that a subnet of a convergent subnet of a net is again a convergent subnet of the original net.

21 Prove that an ultrafilter converges to a point iff that point is a cluster point of it. Use it to show that a topological space is compact iff every ultrafilter in it is convergent.

22 Proving the required propositions show that a subspace of a locally compact, Hausdorff space is locally compact iff it is open in its closure.

(3 × 5 = 15)