QP CODE: 24045341 24045341 **Reg No** : ....

Name : ....

M.Sc DEGREE (CSS) EXAMINATION, DECEMBER 2024

First Semester

CORE - ME010103 - BASIC TOPOLOGY

M.Sc MATHEMATICS,M.Sc MATHEMATICS(SF)

2019 ADMISSION ONWARDS

D33CA44C

Time: 3 Hours Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Define a metric space on a non empty set. Give an example of a metric which is translation invariant.
2. Define a Sierpinskis space. What is its peculiarity?
3. Define subspace of a topological space. Give example of two topologies on X such that for a subset Y of X ,the subspace topology on Y and its complement X-Y are the same.
4. Define Closed set, Clopen set and Dense set of a topological space.
5. Define neighborhood of a point and interior point of a set.
6. Define homeomorphism.
7. Let X be a Lindeloff space and A be a closed subset of X . Prove that A in its relative topology is Lindeloff.
8. Let X be a space and C be a connected subset of X. Suppose $C \subset A \cup B$ where A and B are mutually separated subsets of X. Prove that *either* $C \subset A$ *or* $C \subset B$.
9. Give an example of a normal space which is not regular.
10. Write down the equivalent conditions of normality.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Show that for a second countable space every open cover of it has a countable subcover
12. Let (X,τ) be a topological space. Show that $S \subset \tau$ is a sub base for τ if and only if S generate the topology τ

13. Let $\{f_i : X \to Y_i / i \in I\}$ be a family of functions from a set X to a family of topological spaces $\{(Y_i, \mathfrak{J}_i) / i \in I\}$. Show that there exists a unique smallest topology $\mathfrak{J}$ on X which makes each f_i continuous.

14. *Let $f : X \to Y$ be a quotient map, where X, Y are topological spaces. Then prove that a subset A of Y is open in the quotient topology if and only if $f^{-1}(A)$ is open in X.*

15. Define a second countable space and a first countable space .Prove the relation between them.

16. Prove that closure of a connected subset is connected

17. *Show that complement of a countable set in R^2 is connected*

18. *Give and establish four equivalent conditions for T_1 space*

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any ***two*** *questions.*

Weight ***5*** *each.*

19. Define convergence of sequence in a topological space. How convergence of sequence behaves in Indiscrete ,Discrete and Cofinite topologies on a set X?

20. Let $f : X \to Y$ be a function , where $\mathfrak{J}, \mathcal{U}$ be topologies on X, Y respectively. Then prove that the following statements are equivalent.(i) f is continuous. (ii) for all $V \in \mathcal{U}, f^{-1}(V) \in \mathfrak{J}$. (iii) for any closed subset A of Y, $f^{-1}(A)$ is closed in X. (iv) for all A $\subseteq$ X, $f(\bar{A}) \subseteq \overline{f(A)}$.

21. (i) Let (X, τ) be a topological space and A be a subset of X. Prove that A is a Lindeloff subset of X if and onlyif the space $(A, \tau_{|A})$ is Lindeloff.

(ii) Prove that every second countable space is Lindeloff.

22. Show that quotient space of a locally connected space is locally connected

(2×5=10 weightage)