QP CODE: 24045345

24045345

Reg No : ....

Name : ....

M.Sc DEGREE (CSS) EXAMINATION, DECEMBER 2024

First Semester

CORE - ME010105 - GRAPH THEORY

M.Sc MATHEMATICS,M.Sc MATHEMATICS(SF)

2019 ADMISSION ONWARDS

A990F523

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Define the complement of a graph and self complementary graph. Give an example of a selfcomplementary graph of order 5.
2. Let G be a simple connected graph with n vertices such that Aut(G) is isomorphic to symmetric group S_n on n symbols. Show that G is a complete graph K_n.
3. Prove that an edge e = xy of a connected graph G is a cut edge of G if and only if there exist vertices u and v such that e belongs to every u-v path in G.
4. Show that a simple graph with ω components is a forest if and only if m = n - ω .
5. Write Kruskal's algorithm for determining a minimum weight spanning tree in a connected weighted graph.
6. Define closure of a graph. Give an example.
7. Define chromatic number and chromatic partition of vertices of a graph G.
8. For a simple graph G, prove that $\chi(\overline{G}) \geq \alpha(G)$
9. *Show that a planar graph with minimum degree at least 5 contains at least 12 vertices.*
10. *Define self dual of a plane graph with example*

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. a) Prove in a simple graph G with six vertices either G or $\overline{G}$ contains a triangle.

    b) Show that in any group of n persons $(n \geq 2)$ there are at least two with the same number of friends.

12. Find the order and size of $G_1 \vee G_2$

13. Prove for any loopless connected graph G, $\kappa(G) \leq \lambda(G) \leq \delta(G)$

14. Prove that if C is any cycle of a simple block G, then there exists a sequence of nonseparable subgraphs C = $B_0, B_1, B_2, \ldots\ldots B_r$ =G such that B_{i+1} is an edge-disjoint union of B_i and a path P_i, where the only vertices common to B_i and P_i are the end vertices of P_i, $0 \leq I \leq r-1$.

15. Let G be asimple graph with $n \geq 3$ vertices. if $d(u) + d(v) \geq n - 1$ for every pair of non adjacent vertices u and v of G, then prove that G is traceable.

16. Prove that the 3-critical graphs are just the odd cycles C_{2n+1}.

17. *Show that a graph is planar if and only if it is embeddable on a sphere.*

18. Write the adjacency matrix of the Petersen graph

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. (a) Define and give examples

i. Dicomponent of a graph
ii. Strict graph
iii. Symmetric graph

(b) Prove Moon's theorem and show that a tournament is diconnected if and only if it has a spanning directed cycle.

20. State and prove Cayley's formula by proving all necessary lemmas.

21. (a) Explain Konigsberg Bridge Problem. Draw the graph associated with this. Is this graph Eulerian?. Justify the claim.
(b) Prove that a non trivial connected graph G is eulerian if and only if it is an an edge disjoint union of cycles.

22. *Prove that every planar graph is 5 – vertex colorable.*

(2×5=10 weightage)