

QP CODE: 25010008



Reg No :

Name :

B.Sc DEGREE (CBCS) SPECIAL REAPPEARANCE EXAMINATIONS, FEBRUARY 2025
Fifth Semester

CORE COURSE - MM5CRT03 - ABSTRACT ALGEBRA

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2022 Admission Only

CCDF9CD1

Time: 3 Hours

Max. Marks : 80

Part A

Answer any **ten** questions.

Each question carries **2** marks.

1. Check whether usual multiplication is a binary operation on the set \mathbb{C} .
2. Write two examples for non structural property of a binary structure $\langle S, * \rangle$.
3. Define order of a group.
4. Define the **symmetric group on n letters**. What is its order?
5. Define the **left regular representation** of a group G .
6. Define **even** and **odd** permutations. Give examples.
7. Define the **alternating group A_n on n letters**. What is its order?
8. Check whether $f : (\mathbb{R}, +) \rightarrow (\mathbb{Z}, +)$ defined by $f(x) = [x]$, the greatest integer $\leq x$ is a group homomorphism or not.
9. Show that S_n is not a simple group when $n \geq 3$.
10. Define a) a commutative ring b) a ring with unity
11. Let R be a commutative ring with unity of characteristic 4. Compute and simplify $(a + b)^4$ for $a, b \in R$
12. Mark each of the following true or false.
 - a) A ring homomorphism $\phi : R \rightarrow R'$ carries ideals of R into ideals of R'
 - b) A ring homomorphism is one to one if and only if the kernel is $\{0\}$

(10×2=20)

Part B

Answer any **six** questions.

Each question carries **5** marks.

13. Prove that $\langle \mathbb{Q}^+, * \rangle$ is a group, where $*$ is defined by $a * b = ab/2$.
14. Find the quotient q and remainder r when 38 is divided by 7 according to the division algorithm.
15. Find all orders of subgroups of the group \mathbb{Z}_6 .
16. Prove that every permutation σ of a finite set is a product of disjoint cycles. Express the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 6 & 4 & 1 & 8 & 2 & 5 & 7 \end{pmatrix}$ in S_8 as a product of transpositions.
17. State and prove the **theorem of Lagrange**.
18. Show that composition of group homomorphisms is again a group homomorphism.
19. Prove that the factor group of cyclic group is cyclic.
20. Prove that $M_n(R)$ is a ring where $M_n(R)$ is the collection of all $n \times n$ matrices having elements of R as entries.
21. Let N be an ideal of a ring R . Prove that $\gamma: R \rightarrow R/N$ given by $\gamma(x) = x + N$ is a ring homomorphism with kernel N .

(6×5=30)

Part C

Answer any **two** questions.

Each question carries **15** marks.

22. Let G be a group with binary operation $*$. Then prove the following:
- The left and right cancellation laws hold in G .
 - The linear equations $a * x = b$ and $y * a = b$ have unique solutions x and y in G , where a and b are any elements of G .
- 23.
- Let H be a subgroup of a group G . Let the relation \sim_R be defined on G by $a \sim_R b$ if and only if $ab^{-1} \in H$. Then show that \sim_R is an equivalence relation on G . What is the cell in the corresponding partition of G containing $a \in G$?
 - Let H be the subgroup $\langle \mu_1 \rangle = \{\rho_0, \mu_1\}$ of S_3 . Find the partitions of S_3 into left cosets of H , and the partition into right cosets of H .
24. Let H be a subgroup of a group G . prove that $aHbH = abH$ defines a binary operation on G/H if and only if H is a normal subgroup of G . Then further show that if H is a normal subgroup of a group G then G/H is a group, under the binary operation $aHbH = abH$.
- 25.
- Prove that the divisors of 0 in \mathbb{Z}_n are those nonzero elements that are not relatively prime to n .
 - Find the divisors of \mathbb{Z}_{16} .
 - Prove that \mathbb{Z}_p , where p is prime has no divisors of 0.

(2×15=30)