25020817

QP CODE: 25020817

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE / MERCY CHANCE EXAMINATIONS, FEBRUARY 2025

Sixth Semester

CORE COURSE - MM6CRT04 - LINEAR ALGEBRA

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

98A2D245

Time: 3 Hours

Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. *Prove that A' is an orthogonal matrix if A is an orthogonal nxn matrix.*
2. a)Define linearly dependent rows.

   b)*Prove that in the matrix* $A = \begin{bmatrix} 1 & 2 & 0 & 2 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ *the columns are linearly dependent.*
3. *Prove that X= { (x,0) : x ε R} is a subspace of the vector space* R^2
4. *Define span S of a vector space V and Prove that S = {(1,0), (0,1)} is a spanning set of* R^2
5. Prove that { (1,1,1), (1,2,3) , (2,-1, 1) } is a basis of R^3.
6. If $f : V \to W$ is linear, X is a subset of V and Y is a subset of W, define direct image of X under f and inverse image of Y under f.
7. If $f : \mathbb{R}^2 \to \mathbb{R}^2$ is given by $f(a,b) = (b,0)$, prove that $Im\, f = Ker\, f$.
8. If V and W are vector spaces of the same dimension n over F, then prove that V and W are isomorphic.
9. Define a nilpotent linear mapping f on a vector space V of dimension n over a field F. What is meant by index of nilpotency of f.
10. Define eigen value and eigen vector of a matrix.
11. Define eigen value of a linear map and the eigen vector associated with it.
12. Define diagonalizable linear map and diagonalizable matrix.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. a)Prove that every square matrix can be expressed uniquely as the sum of a symmetric matrix and a skew symmetric matrix.

b)If $A = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$ Prove that $A^n = \begin{bmatrix} cosn\theta & sinn\theta \\ -sinn\theta & cosn\theta \end{bmatrix}$

14. Show that the system of equations x + 2y + 3z + 3t = 3 , x + 2y + 3t = 1 , x + z + t = 3, x + y + z + 2t = 1 has no solution.

15. Prove that Rn[x] be the set of polynomials of degree atmost n with real coefficients is a real vector space.

16. If S is a subset of V, then prove that S is a basis if and only if S is a minimal spanning set.

17. Show that the linear mapping $f : \mathbb{R}^3 \to \mathbb{R}^3$ given by $f(x, y, z) = (x + y + z,\ 2x - y - z,\ x + 2y - z)$ is both surjective and injective.

18. Consider the linear mapping $f : \mathbb{R}^3 \to \mathbb{R}^2$ given by $f(x, y, z) = (2x - y,\ 2y - z)$. Determine the matrix of f

(1) relative to the natural ordered bases.

(2) relative to the ordered bases $\{(1, 1, 1), (0, 1, 1), (0, 0, 1)\}$ and $\{(0, 1), (1, 1)\}$.

19. Define similar matrices. Prove that the relation of being similar is an equivalence relation on the set of $n \times n$ matrices.

20. Find the eigen values and eigen vectors of $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$

21. For the nXn tridiagonal matrix An = $\begin{bmatrix} 2 & 1 & 0 & 0 & \dots & 0 & 0 \\ 1 & 2 & 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & 2 & 1 & \dots & 0 & 0 \\ . & . & . & . & ..... & . & . \\ 0 & 0 & 0 & 0 & \dots & 2 & 1 \\ 0 & 0 & 0 & 0 & \dots & 1 & 2 \end{bmatrix}$ Prove that det An = n + 1.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. *a) Reduce the following matrix to row echelon form* $\begin{bmatrix} 1 & 2 & 0 & 3 & 1 \\ 1 & 2 & 3 & 3 & 3 \\ 1 & 0 & 1 & 1 & 3 \\ 1 & 1 & 1 & 2 & 1 \end{bmatrix}$

b) Prove that by using elementary row operation, a non-zero matrix can be transformed to a row-echelon matrix.

c) Prove that every non-zero matrix A can be transformed to a Hermite matrix by using elementary row operations.

23. a)Define a left inverse and right inverse of a matrix.

b) Prove that the matrix A= $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 3 & 3 \end{bmatrix}$ has a common unique left inverse and unique right inverse.

c)Find the inverse of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix}$

d)If $A_1, A_2, \ldots, A_P$ are invertible nxn matrices. Prove that the product $A_1, A_2, \ldots A_P$ is invertible and that $(A_1 A_2 \ldots A_P)^{-1} = A_P^{-1} \ldots A_2^{-1} A_1^{-1}$

24. Let V and W be vector spaces each of dimension n over a field F. If $f : V \to W$ is linear then prove that the following statements are equivalent:
(i) f is injective (ii) f is surjective (iii) f is bijective (iv) f carries bases to bases, in the sense that if $\{v_1, \ldots, v_n\}$ is a basis of V then $\{f(v_1), \ldots, f(v_n)\}$ is a basis of W.

25. Consider the linear mapping $f : \mathbb{R}^3 \to \mathbb{R}^3$ given by $f(x, y, z) = (y, -x, z)$. Compute the matrix A of f relative to the natural ordered basis and the B matrix of f relative to the ordered basis $\{(1, 1, 0), (0, 1, 1), (1, 0, 1)\}$. Determine an invertible matrix X such that $A = X^{-1}BX$.

(2×15=30)