25019639

QP CODE: 25019639

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS)) REGULAR/ IMPROVEMENT/ REAPPEARANCE / MERCY CHANCE EXAMINATIONS, FEBRUARY 2025

Fourth Semester

Core Course - MM4CRT01 - VECTOR CALCULUS, THEORY OF NUMBERS AND LAPLACE TRANSFORMS

(Common for B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science)

2017 Admission Onwards

ECE842DB

Time: 3 Hours | Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Find the component equation and simplified component equation for the plane through $P_0(-3,0,7)$ perpendicular to $\mathbf{n}=5\mathbf{i}+2\mathbf{j}-\mathbf{k}$.
2. Give the parametrization of a helix.
3. Find the arc length parameter along the helix $\mathbf{r}(t)=(\cos t)\mathbf{i}+(\sin t)\mathbf{j}+t\mathbf{k}$ from t_0 to t.
4. Define gradient vector of a scalar function $f(x,y,z)$.
5. Define Curl of a vector field F.
6. Find the divergence of the vector field $F=xi+yj+zk$.
7. Check whether the following set S of integers constitute a complete set of residues modulo 7 or not:
   $S=\{-12,-4,11,13,22,82,91\}$.
8. State Wilson's theorem.
9. Define Euler phi-function with example.
10. Find $\mathscr{L}(t^2+3)^2$.
11. Find $\mathscr{L}^{-1}\left\{\frac{1}{s^2+\frac{s}{2}}\right\}$.

12. Evaluate $\mathscr{L}^{-1}\left[\frac{1}{(s^2+\omega^2)^2}\right]$ using convolution theorem.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Represent the directional derivative of a differentiable function in the plane as a dot product.
Also, find the directions in which $f(x, y) = (x^2/2) + (y^2/2)$

    1. increases most rapidly at the point $(1, 1)$.
    2. decreases most rapidly at $(1, 1)$.
    3. having zero change in f at $(1, 1)$.

14. Define the **tangent plane** and the **normal line** at a point on a smooth surface in space.
Find the plane tangent to the surface $z = x\cos y - ye^x$ at $(0, 0, 0)$.

15. Evaluate the line integral $\int(2xcosy)dx - (x^2 siny)dy$ along the parabola $y = (x-1)^2$ from $(1, 0)$ to $(0, 1)$.

16. Find a parametrization of the cylinder $(x-3)^2 + y^2 = 9, 0 \le z \le 5$.

17. Integrate $G(x, y, z) = xyz$ over the triangular surface with vertices $(1, 0, 0), (0, 2, 0)$ and $(0, 1, 1)$.

18. Derive the congruence: $a^{13} \equiv a \pmod{3.7.13}$ for all a.

19. Let n be a composite square-free integer, say, $n = p_1 p_2 \ldots p_r$, where the p_i are distinct primes.
If $p_i - 1 | (n-1)$ for $i = 1, 2, \ldots, r$, then prove that n is an absolute pseudoprime.

20. Find $\mathscr{L}^{-1}\left\{\frac{s^2+2s+5}{(s-1)(s-2)(s-3)}\right\}$.

21. Using Laplace Transform, solve $y' - 6y = 0,\ y(2) = 4$.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22.

    1. Find and graph the **osculating circle** of the parabola $y = x^2$ at the origin.
    2. Find the **curvature** for the helix
    $\mathbf{r}(t) = (a\cos t)\mathbf{i} + (a\sin t)\mathbf{j} + bt\mathbf{k}, a, b \ge 0, a^2 + b^2 \ne 0.$

23. State Green's Theorem and apply it to find the counter clockwise circulation and outward flux for the field $F = (x - y)i + (y - x)j$ and the curve C is bounded by the square $x = 0, x = 1, y = 0, y = 1$.

24.
1. State and prove Fermat's theorem.
2. Is the converse of Fermat's theorem is true or false? Give justifications.

25.
1. Using Laplace Transform, solve
$y'' + 2y' + 5y = 50\,t - 150,\ y(3) = -4,\ y'(3) = 14.$
2. Solve the Volterra integral equation of the second kind
$y(t) - \int_0^t y(\tau)\sin(t - \tau)d\tau = t.$

(2×15=30)