QP CODE: 25023022

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Reg No : .....

Name : .....

B.Sc DEGREE (CBCS) IMPROVEMENT / REAPPEARANCE/ MERCY CHANCE EXAMINATIONS, APRIL 2025

Second Semester

Core Course - MM2CRT01 - MATHEMATICS - ANALYTIC GEOMETRY, TRIGONOMETRY AND DIFFERENTIAL CALCULUS

(Common for B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science)

2017 Admission Onwards

EAC587D6

Time: 3 Hours Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Show that two tangents can be drawn from any point to a parabola.
2. Derive the equation of chord of contact of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
3. Prove that the sum of squares of two conjugate semi-diameters of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is a constant.
4. If P and D are the extremities of semi-conjugate diameters of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, show that the locus of the middle point PD is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{1}{2}$.
5. Find the polar coordinates corresponding to the cartesian coordinate (-3, $\sqrt{3}$).
6. Find the polar equation of the conic having the axis of the conic makes an angle α with the initial line.
7. Prove that sin(x+y) = sinx cosy + cosx siny.
8. Prove that sinh(-x) = - sinh x
9. Separate into real and imaginary parts $cos(\alpha + i\beta)$.
10. Find the n^{th} derivative of e^{ax} .

11. Find the n[th] derivative of $cos^3 x$.

12. Evaluate $lim_{x \to \frac{\pi}{2}} (xtanx - \frac{1}{2}\pi secx)$.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Tangents to an ellipse make angles θ_1 and θ_2 with the major axis. Show that the locus of their intersection, when $cot\theta_1 + cot\theta_2 = k^2$ is $y^2k^2 - 2xy = b^2k^2$.

14. Find the orthoptic locus of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

15. Prove that the locus of the pole, with respect to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, of any tangent to the auxiliary circle is $\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{a^2}$.

16. Show that the locus of the poles of normal chords of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is the curve $y^2a^6 - x^2b^6 = (a^2+b^2)^2x^2y^2$.

17. Show that the tangents at the extremities of any focal chord of a conic intersect on the corresponding directrix.

18. Sum the series $sin\alpha + csin(\alpha + \beta) + \frac{c^2}{2!} sin(\alpha + 2\beta) + \ldots$ .where c is less than unity.

19. Sum the series $sinh\alpha - \frac{1}{2} sinh2\alpha + \frac{1}{3} sinh3\alpha - \ldots$

20. If $y = cos(msin^{-1}x)$, show that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$ and hence find $y_n(0)$.

21. Determine $lim[\frac{\pi}{2} - x]^{tanx}$ as $x \to [\frac{\pi}{2} - 0]$.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. Let P be a point of the parabola $y^2 = 4ax$. If PV is the diameter which bisects the chord QR at V, prove that $QV^2 = 4PV.SP$, where S is focus of the parabola.

23. Derive the equation of the tangent to the circle $r = 2a \cos\theta$ at α.

24. Factorize the expression $x^n - 1$

25. (a) Find the third differential coefficient of $e^{ax}cosbx$.

(b) Show that if $y = sin(sin^{-1}x)$, then $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$.

(c) If $y = [x + \sqrt{1+x^2}]^m$, show that $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - m^2y = 0$.

(2×15=30)