19001689

19001689

Reg. No..............................

Name..................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, JUNE 2019

Second Semester

Faculty of Science

Branch I (A)—Mathematics

MT 02 C09—PARTIAL DIFFERENTIAL EQUATIONS

(2012 Admission onwards)

Time : Three Hours | Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question carries 1 *weight.*

1. Find the primitive : $yzdx + xzdy + xydz = 0$.

2. Eliminate a and b from : $2z = (ax + y)^2 + b$.

3. Find the complete integral of $p^2 + q^2 = x + y$.

4. Define Compatible system of first order equations.

5. Find the particular integral $\left(D^2 - D^1\right) Z = e^{2x + y}$.

6. Eliminate the arbitrary function ϕ

$$z = \frac{1}{x}\phi(y - x) + \phi^1(y - x).$$

7. Define equipotential surface.

8. Explain : exterior Dirichlet Problem.

(5 × 1 = 5)

Turn over

Part B

Answer any **five** *questions.*
Each question carries 2 *weight.*

9. Find the general integral $x^2 \, p + y^2 \, q = (x + y) \, z$.

10. Show that the Pfaffian differential equation $yz \, dx + 2xz \, dy - 3xy \, dz = 0$ is integrable and find the corresponding integral.

11. Find the complete integral of $f = z^2 - pqxy = 0$ by Charpit's method.

12. Find the complete integral of $2x\left(z^2 \, q^2 + 1\right) = pz$.

13. Reduce to canonical form $u_{xx} - x^2 \, u_{yy} = 0$.

14. Show that :

$$F\left(D, D^1\right)\left\{e^{ax+by} \, \phi(x, y)\right\} = e^{ax+by} \, F\left(D + a, D^1 + b\right) \phi(x, y).$$

15. Solve by separation of variable :

$$\frac{\partial^2 z}{\partial x^2} = \frac{1}{k} \frac{\partial z}{\partial t}.$$

16. Establish a necessary condition for the existence of the solution of the interior Neumann problem.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question carries 5 *weight.*

17. Verify that the equation :

$$yz(y + z) \, dx + xz(x + z) \, dy + xy(x + y) \, dz = 0$$

is integrable and find its solution.

18. Find the complete integral of $p^2x + qy - z = 0$. and derive the equation of the integral surface containing the line $y = 1, x + z = 0$.

19. Solve by Jacobi's method :

$$p^2x + q^2y = z.$$

20. Find by the method of characteristics, the integral surface of $pq = xy$ which passes through the curve $z = x, y = 0$.

21. Reduce :

$$(n-1)^2 u_{xx} - y^{2n} u_{yy} = n\, y^{2n-1} u_y$$

to a canonical form and solve if possible.

22. Find the solution of :

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{1}{k}\frac{\partial z}{\partial t}$$

by method of separation of variables.

(3 × 5 = 15)