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Reg. No...............................

Name...................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, JUNE 2019

Second Semester

Faculty of Science

Branch I (a)–Mathematics

MT 02 C10—REAL ANALYSIS

(2012 Admission onwards)

Time : Three Hours | Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Prove the additive property of arc-lengths.
2. When a function u is said to define a change of parameter ?
3. Prove that $\int_a^b d\alpha(x) = \alpha(b) - \alpha(a)$, directly from the definition of Riemann-Stieltjes integral.
4. State the additive property for upper Stieltjes integrals.
5. For $m = 1, 2, 3, \ldots$ and for $n = 1, 2, 3$. Let $S_{m,n} = \dfrac{m}{m+n}$ show that $\lim\limits_{n \to \infty} \lim\limits_{m \to \infty} S_{m,n} \neq \lim\limits_{m \to \infty} \lim\limits_{n \to \infty} S_{m,n}$.
6. State Stone-Weierstrass theorem.
7. Explain : Algebraic completeness of complex field.
8. Obtain the periods of the function C and S.

(5 × 1 = 5)

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Define a monotonic function. If f is monotonically decreasing on $[a, b]$. Prove that f is of bounded variation on $[a, b]$.
10. Show that $f(x) = x^2 \cos\left(\dfrac{1}{x}\right)$ if $x \neq 0$ and $f(0) = 0$ is of bounded variation on [0, 1].

Turn over

11. Suppose f is monotonic on $[a, b]$ and if α is continuous on $[a, b]$. Prove that f is Riemann-Stieltjes integrals on $[a, b]$.

12. State and prove additive property of Riemann-Stieltjes integrals.

13. Define uniform convergence of a sequence of function f_n on a set E to a function f. If $\{f_n\}$ and $\{g_n\}$ converge uniformly on a set E, show that $\{f_n + g_n\}$ also converge uniformly on E.

14. Let $f_n(x) = x$ for all $x \in E$ and all n and let $g_n(x) = \frac{1}{n}$ for all $x \in E$ and all n. Examine the uniform convergence of $\{f_n\}, \{g_n\}$ and $\{f_n g_n\}$.

15. Evaluate $\lim_{x \to 0} \frac{e - (1+x)^{1/x}}{x}$.

16. Prove that $\Sigma \frac{1}{p}$ diverges ; the sum extends over all primes.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. Define total variation. Establish additive property of total variation.

18. (a) State and prove fundamental theorem of calculus.

    (b) State and prove integration by parts theorem on R – S integrals.

19. (a) Write sufficient condition for Riemann Stieltjes integrable and prove.

    (b) Establish linearity property of Riemann Stieltjes integrals.

20. If f is continuous on [0, 1] and if :

    $\int_0^1 f(x)\, x^n dx = 0 \ (n = 0, 1, 2, \ldots)$. Prove that $f(x) = 0$ on [0, 1].

21. Give example to show that uniform convergence of $\{f_n\}$ implies nothing about the sequence $\{f_n^1\}$.

    State the hypotheses required for the assertion that $f_n^1 \to f^1$ if $f_n \to f$ and prove.

22. Define the exponential, logarithmic and trigonometric functions. Establish their properties and describe the relationship between them.

(3 × 5 = 15)