**QP CODE: 19002353**

19002353

**Reg No** : ....................

**Name** : ....................

## M.Sc. DEGREE (C.S.S ) EXAMINATION, NOVEMBER 2019

## First Semester

Faculty of Science

MATHEMATICS

## Core - ME010102 - LINEAR ALGEBRA

2019 Admission Onwards

729B2BCA

Time: 3 Hours

Maximum Weight :30

**Part A (Short Answer Questions)**

*Answer any **eight** questions.*

*Weight **1** each.*

1. Define vector space. Is $\mathbb{R}$ a vector space over $\mathbb{C}$ ?
2. Prove that if two vectors are linearly dependent, one of them is a scalar multiple of the other.
3. Find two linear operators $T$ and $U$ on $R^2$ such that $UT = 0$ , but $TU \neq 0$.
4. Prove that if F is any field then the n-tuple space $F^n$ and the space $F^{n\times 1}$ of all $n \times 1$ matrices are isomorphic.
5. Define dual space and double dual space of a vector space.
6. Prove that a linear combination of n-linear functions is n-linear.
7. Let D be a 2-linear function with the property that D(A) = 0 for all 2 x 2 matrices A over K having equal rows. Then show that D is alternating.
8. Use Cramer's Rule to solve
   x + 2y + 3z = 17
   3x + 2y + z = 11
   x − 5y + z = −5.
9. Find the characteristic values, if any, of the linear operator $T$ on $\mathbb{R}^2$ which is represented in the standard ordered basis by the matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
10. Let $V$ be a finite dimensional vector space and let $W_1, \cdots, W_k$ be subspaces of $V$ such that $V = W_1 \oplus \cdots \oplus W_k$ . Prove that any vector $\alpha \in V$ can be uniquely represented as a sum $\alpha = \alpha_1 + \cdots + \alpha_k$ where $\alpha_i \in W_i$

(8×1=8 weightage)

**Part B (Short Essay/Problems)**

*Answer any **six** questions.*

*Weight **2** each.*

11. Suppose P is an n x n invertible matrix over F. Let V be an n-dimensional vector space over F, and let $\mathcal{B}$ be an ordered basis of V. Then show that there is a unique ordered basis for V such that (i) $[\alpha]_{\mathcal{B}} = P[\alpha]_{\mathcal{B}'}$ and (ii) $[\alpha]_{\mathcal{B}'} = P^{-1}[\alpha]_{\mathcal{B}}$ for every vector $\alpha$ in V.

12. Show that row-equivalent matrices have same row space.

13. If $A \in F^{m\times n}$ prove that $row\ rank(A) = column\ rank(A)$.

14. If T is a linear on $R^3$ defined as $T(x_1, x_2, x_3) = (3x_1 + x_3, -2x_1 + x_2, -x_1 + 2x_2 + 4x_3)$ ,find the matrix of T in the standard ordered basis of $R^3$ .

15. Let V and W be finite dimensional vector spaces over the field F and $T : V \to W$ is a linear transformation. Prove that $Range\ T^t$ is the annihilator of the null space of $T$.

16. If A is a 2 x 2 matrix over a field, prove that det (I+A)= 1 + det A if and only if trace(A) = 0.

17. Let $T$ be a linear operator on $\mathbb{R}^3$ which is represented in the standard ordered basis by the matrix $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$. Find an invertible matrix $P$ such that the matrix $D = P^{-1}AP$ is a diagonal matrix.

18. Let $a, b$ and $c$ be elements of a field $F$ and $A = \begin{bmatrix} 0 & 0 & c \\ 1 & 0 & b \\ 0 & 1 & a \end{bmatrix}$ be the matrix over $F$. Prove that the characteristic and minimal polynomial for $A$ is $x^3 - ax^2 - bx - c$

(6×2=12 weightage)

**Part C (Essay Type Questions)**

*Answer any **two** questions.*

*Weight **5** each.*

19. (a) Let W be the set of all (x1, x2, x3, x4, x5) in $\mathbb{R}^5$ which satisfy

$2x1 - x2 + \frac{4}{3}x3 - x4 = 0$

$x1 + \frac{2}{3}x3 - x5 = 0$

$9x1 - 3x2 + 6x3 - 3x4 - 3x5 = 0.$

Find a finite set of vectors which spans W.

(b) Let V be the vector space of all functions from $\mathbb{R}$ into $\mathbb{R}$; let Ve be the subset of even functions and let Vo be the subset of odd functions.

(i) Prove that Ve and Vo are subspaces of V.

(ii) Prove that Ve + Vo = V.

(iii) Prove that Ve ∩ Vo = {0}.

20. Let $V$ be a finite dimensional vector space over the field F. For any subspace $W$ of $V$ prove that $dimW + dimW^0 = dimV$

21. Let K be a commutative ring with identity and let n be a positive integer. Then prove that there is precisely one determinant function on the set of n x n matrices over K, and it is the function defined by det (A)= $\sum_{\sigma} sgn(\sigma)A(1,\sigma_1) \ldots A(n,\sigma_n)$. If D is any alternating n-linear function on Knxn , then prove that for each n x n matrix A, D(A) =(det A)D(I).

22.

1. Let $V$ be a finite dimensional vector space over the field $F$ and let $T$ be a linear operator on $V$. Prove that $T$ is triangulable if and only if the minimal polynomial for $T$ is a product of linear polynomials over $F$
2. Is the marix $A$ similar over the field of real numbers to a triangular matrix where $A = \begin{bmatrix} 0 & 1 & 0 \\ 2 & -2 & 2 \\ 2 & -3 & 2 \end{bmatrix}$

(2×5=10 weightage)