QP CODE: 20100168

Reg No : ....

Name : ....

BSc DEGREE (CBCS) EXAMINATION, FEBRUARY 2020

Fifth Semester

Core Course - MM5CRT03 - ABSTRACT ALGEBRA

B.Sc Mathematics Model I,B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

8D08D53F

Time: 3 Hours | Maximum Marks :80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Check whether usual multiplication is a binary operation on the set $\mathbb{C}$.
2. State whether the set $\mathbb{Z}^+$ under multiplication is a group. Justify.
3. Define order of an element in a group.
4. Show that the *permutation multiplication* is a binary operation on the collection of all permutations of a set A.
5. Define the **right regular representation** of a group G.
6. Find all orbits of the permutation $\sigma : \mathbb{Z} \to \mathbb{Z}$ where $\sigma(n) = n + 2$.
7. Show that every coset (left or right) of a subgroup H of a group G has the same number of elements as H.
8. Check whether $f : (\mathbb{R}, +) \to (\mathbb{Z}, +)$ defined by $f(x) = \lfloor x \rfloor$, the greatest integer $\leq x$ is a group homomorphism or not.
9. Show that S_n is not a simple group when n$\geq$ 3.

10. Show that the matrix $\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$ is a divisor of zero in $M_2(Z)$

11. Prove that Z_p is a field if p is a prime

12. Give an example to show that a factor ring of an integral domain may have divisors of 0

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Prove that if $\phi: S \to S'$ is an isomorphism of $\langle S, * \rangle$ with $\langle S', *' \rangle$, and e is the identity element for $*$ on S,
then $\phi\ (e\ )$ is an identity element for $*'$ on S' .

14. Prove that a subset H of a group G is a subgroup of G if and only if
a) H is closed under the binary operation of G,
b) the identity element e of G is in H,
c) for all $a \in H$ it is true that $a^{-1} \in H$ also.

15. Let G be a group and let $a \in G$. Then prove that $H = \{a^n / n \in \mathbb{Z}\}$ is a subgroup of G and is the smallest subgroup of G that contains a.

16. Prove from linear algebra that no permutation in S_n can be expressed both as a product of an even number of transpositions and as a product of an odd number of transpositions.

17. If $n \geq 2$, then prove that the collection of all even permutations of $\{1, 2, 3, \cdots, n\}$ forms a subgroup of order $\frac{n!}{2}$ of the symmetric group S_n.

18. Upto isomorphism find S_n / A_n

19. Show that A_n is a normal subgroup of S_n

20. Check whether Z^+ wth the usual addition and multiplication is a ring

21. Show that $a^2 - b^2 = (a + b)(a - b)$ for all a and b in a Ring R If and only if R is commutative.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. Find all subgroups of $\mathbb{Z}_{36}$ and draw the subgroup diagram.

23.
    1. Let H be a subgroup of a group G. Let the relation $\sim_L$ be defined on G by $a \sim_L b$ if and only if $a^{-1}b \in H$. Then show that $\sim_L$ is an equivalence relation on G. What is the cell in the corresponding partition of G containing $a \in G$?
    2. Let H be the subgroup $< \mu_1 >= \{\rho_0, \mu_1\}$ of S_3. Find the partitions of S_3 into left cosets of H, and the partition into right cosets of H.

24. Let H be a subgroup of a group G. prove that $aHbH = abH$ defines a binary operation on G/H if and only if H is a normal subgroup of G. Then furthere show that if H is a normal subgroup of a group G then G/H is a group. under the binary operation $aHbH = abH$.

25. a) Show that $I_a = \{x \in R/ax = 0\}$ is an ideal of R, R is a commutative ring and $a \in R$

    b) Show that an intersection of ideals of a ring R is again an ideal of R

    c) Find all ideals N of Z_{12}

(2×15=30)