QP CODE: 20100166 20100166 **Reg No** : ....

Name : ....

BSc DEGREE (CBCS) EXAMINATION, FEBRUARY 2020

Fifth Semester

Core Course - MM5CRT01 - MATHEMATICAL ANALYSIS

B.Sc Computer Applications Model III Triple Main ,B.Sc Mathematics Model I,B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

E15905D3

Time: 3 Hours Maximum Marks :80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Prove that $z + a = a, \forall z, a \in R$ then $z = 0$?
2. Define ϵ neighbourhood of a point? If x belongs to $V_\epsilon(a)$ for all $\epsilon > 0$ then prove that $x = a$?
3. If $t > 0$ prove that there exist an $n_t \in N$ such that $0 < \dfrac{1}{n_t} < t$
4. Justify the validity of the following statement with proper reasoning "A positive real number is rational then its decimal expansion is periodic"
5. Find $lim(\frac{\sqrt{n}-1}{\sqrt{n}+1})$.
6. Define monotone increasing and monotone decreasing sequences. Give examples.
7. Prove that if a sequence X = (x_n) converges to x, then every subsequence of X also converges to x.
8. Prove that every Cauchy sequence of real numbers is bounded.
9. If c > 1, prove that lim $(c^n) = +\infty$.
10. Give an example of a convergent series of Real Numbers which is not absolute convergent.

11. Test the convergence of $\sum_{1}^{\infty} \frac{(-1)^{n+1}}{(n^2+1)}$

12. Define the right-hand and left-hand infinite limits.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. State and prove three equivalent definitions for a countable set ?

14. Let S be a nonempty subset of real numbers that is bounded below, Prove that $Inf\ S = -Sup\{-s : s \in S\}$?

15. Prove that $lim(\frac{1}{n^2+1}) = 0.$

16. Prove that lim $(n^{1/n}) = 1$.

17. Let X = (x_n) and Y = (y_n) be sequences of real numbers that converges to x and y respectively. Prove that the sequences X.Y converges to xy.

18. State and prove the comparison test for the convergence of series. Using this test, show that $\sum_{1}^{\infty} \frac{1}{n^2+n}$ is convergent.

19. State and prove the root test for the absolute convergence of a series in R.

20. Show that $\lim_{x \to 0} \sin(\frac{1}{x})$ does not exist in $\mathscr{R}$.

21. Let f, g be defined on $A \subseteq \mathscr{R}$ and c be a cluster point of A.If f is bounded on a neighborhood of c and $\lim_{x \to c} g = 0$, then prove that $\lim_{x \to c} fg = 0$.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. (a.) State and Prove Nested interval property?
(b.) Prove that the set of real numbers is not countable?

23. (a) State and prove Monotone Convergence Theorem.
(b) Prove that for any real number a > 0, there exists a sequence (s_n) of real numbers that converges to $\sqrt{a}$.

24.

1. When an alternating series can be convergent? Give an example of an alternating convergent series.
2. State and prove Abel's Test.

25. (a) Let $A \subseteq \mathscr{R}$, $f, g : A \to \mathscr{R}$, and let $c \in \mathscr{R}$ be a cluster point of A, Suppose that $f(x) \leq g(x)$ for all $x \in A$, $x \neq c$, Then prove the following

- If $\lim_{x\to c} f = \infty$, then $\lim_{x\to c} g = \infty$.
- If $\lim_{x\to c} g = -\infty$,then $\lim_{x\to c} f = -\infty$.

(b) Give an example of a function that has a right-hand limit but not a left-hand limit at a point.

(c) Evaluate the limit or show that it do not exist " $\lim_{x\to 1} \frac{x}{x-1}$ where $x \neq 1$.

(2×15=30)