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Reg. No.................................

Name....................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, NOVEMBER 2019

Third Semester

Faculty of Science

Branch I : (A) Mathematics

MT03C11—MULTIVARIATE CALCULUS AND INTEGRAL TRANSFORMS

(2012—2018 Admissions)

Time : Three Hours | Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question carries a weight of 1.

1. Explain the matrix of a linear function.
2. Give example to show that existence of all directional derivatives at a point fail to imply continuity.
3. Define periodic function with an example. Also define Fourier Integral.
4. Define Convolution of two functions with example.
5. Give example to show that two mixed partials $D_{1,\ 2}f$ and $D_{2,\ 1}f$ need not be equal.
6. State inverse function theorem.
7. State the theorem on partitions of unity.
8. State the transformation properties of differential forms.

(5 × 1 = 5)

Part B

Answer any **five** *questions.*
Each question carries a weight of 2.

9. Establish the Weierstrass Approximation Theorem.
10. Obtain an integral representation for the arithmetic means of the partial sums of a Fourier Series.
11. State and prove Mean Value Theorem.
12. With usual notations prove :

    m (SoT) = m (S) m (T).

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13. Use Taylor's formula to express :

$$f(x, y) = x^3 + y^3 + xy^2 \text{ in powers of } x - 1 \text{ and } y - 2.$$

14. Find and classify the extreme values (if any) of the function $f(x, y) = (x-1)^2 + (x-y)^4$.

15. If Σ is a 2-surface in R^3 given by $\Sigma(u, v) = (\sin u, \cos v, \sin u \sin v, \cos v)$ $0 \le u \le \pi, 0 \le v \le 2\pi$, show that $\partial\Sigma = 0$.

16. For every $f \in \mathscr{C}(I^k)$, prove $L(f) = L'(f)$.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question carries a weight of 5.

17. State and prove convolution theorem for Fourier Integral Transform.

18. Let f and g be functions from R^n to R^m Assume f is differentiable at C, that $f(c) = 0$ and that g is continuous at C. Let $h(x) = g(x) . f(x)$. Prove that h is differentiatiable at C and, that :

$$h'(c)(u) = g(c) \cdot \{f'(c)(u)\}.$$

19. Suppose T is a $\mathscr{C}'$-mapping of an open set $E \subset R^n$ into an open set $V \subset R^m$, ϕ is a k-surface in E, and ω is a k-form in V. Prove :

$$\int_{T\phi} w = \int_{\phi} w_T$$

20. Show that Cauchy-Riemann equations, along with differentiability of u and v, imply existence of $f'(c)$.

21. Prove : If both partial derivative $D_r f$ and $D_k f$ exist in an n–ball B (c) and if both $D_{r, k} f$ and $D_{k, r} f$ are continuous at c, then

$$D_{r, k} f(c) = D_{k, r} f(c).$$

22. Show that a function with continuous partial derivatives is locally one-to-one near a point where the Jacobian determinant does not vanish.

(3 × 5 = 15)