19002008

19002008

Reg. No.................................

Name....................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, NOVEMBER 2019

Third Semester

Faculty of Science

Branch I (A)—Mathematics

MT 03 C12—FUNCTIONAL ANALYSIS

(2012—2018 Admissions)

Time : Three Hours | Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question carries a weight of 1.

1. Give an example of a normed linear space which is not Banach. Justify your result.
2. Define convex and non-convex sets with example.
3. Define : (a) Annihilator ; (b) Linear functional.
4. Obtain necessary and sufficient conditions for $\|x+y\|^2 = \|x\|^2 + \|y\|^2$.
5. Establish the characterisation of sets in Hilbert space whose span is dense.
6. Show that every orthonormal set in a separate Hilbert space is countable.
7. Write Zorn's lemma and two of its applications.
8. Show that a norm on a vector space X is a sub-linear functional on X.

(5 × 1 = 5)

Part B

Answer any **five** *questions.*
Each question carries a weight of 2.

9. Show that if a normed space has a Schauder basis, it is separable.
10. Prove that every finite dimensional subspace of a normed space is complete.
11. State and prove Riesz lemma.
12. Show that the dual of l' is l^{∞}.

Turn over

13. Obtain necessary and sufficient condition for two Hilbert space to be isomorphic.

14. State four properties of adjoint of an operator and prove them.

15. Define reflexive space with an example prove every Hilbert space is reflexive.

16. Establish Hahn Banach Theorem for normed spaces.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question carries a weight of 5.

17. (a) Define the norm of an separator and obtain an alternate formula for it. Also prove $\| T_1, T_2 \| \leq \| T_1 \| \; \| T_2 \|$ and $\left\| T^n \right\| \leq \left\| T^n \right\|$.

    (b) Prove : T is continuous if and only if T is bounded.

18. (a) Obtain polarization inequality.

    (b) Derive Appolonius identity.

19. (a) Establish the continuity of inner product.

    (b) Obtain Schwarz inequality. Also obtain necessary and sufficient condition for equality.

20. (a) Show that a subspace y of a Hilbert Space H is closed in H if and only if $y = y^{\perp\perp}$.

    (b) Let A and $B \supset A$ be non-empty subsets of an inner product space X. Show that $B^{\perp} \subset A^{\perp}$ and $A^{\perp\perp\perp} = A^{\perp}$.

21. (a) If S and T are normal operators satisfying ST* = T*S and TS* = S*T show that S + T and ST are normal.

    (b) State five basic properties of unitary operators and prove.

22. (a) Establish uniform boundedness theorem.

    (b) Find the Fourier series of :

$$x(t) = \begin{cases} 0 & \text{if } -\pi \leq t < 0 \\ 1 & \text{if } \quad 0 \leq t < \pi \end{cases}$$

and $x(t + 2\pi) = x(t)$.

(3 × 5 = 15)