QP CODE: 19102567

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Reg No : ....

Name : ....

BSc DEGREE (CBCS) EXAMINATION, OCTOBER 2019

Fifth Semester

Core Course - MM5CRT03 - ABSTRACT ALGEBRA

B.Sc Mathematics Model I,B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

D91C9994

Maximum Marks: 80 Time: 3 Hours

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. State homomorphism property of a binary algebraic structure.
2. Define trivial subgroup and non trivial subgroup of a group G .
3. Define generator for a group.
4. Find the number of elements in the set $\{\sigma \in S_5 | \sigma(2) = 5\}$.
5. Find the orbits of the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 8 & 6 & 7 & 4 & 1 & 5 & 2 \end{pmatrix}$ in S_8.
6. Show that any permutation of a finite set of at least two elements is a product of transpositions.Write the identity permutation in S_n for $n \geq 2$ as a product of transpositions.
7. Define the **alternating group** A_n **on** n **letters**. What is its order?
8. Let G be a group. If $\phi : G \to G$ defined by $\phi(g) = g^-1$ is a group homomorphism, show that G is Abelian.
9. If $\phi : G \to G'$ is a group homomorphism and $g \in G$, show that $\phi(g^{-1}) = (\phi(g))^{-1}$.
10. Compute the product in the given ring a) (11) (-4) in Z_{15} b) (16) (3) in Z_{32}
11. Check whether Z is a field .
12. Prove that nZ is an ideal of the ring Z.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Determine whether $*$ defined on $\mathbb{Z}$ by $a * b = a - b$ is a) commutative b) associative.

14. Define a group. Give an example.

15. a) When can we say that two positive integers are relatively prime?
    b) Prove that if r and s are relatively prime and if r divides sm, then r must divide m.

16. Exhibit the left cosets and the right cosets of the subgroup $3\mathbb{Z}$ of $\mathbb{Z}$.

17. State and prove the theorem of Lagrange.

18. Let **G** be a group. Show that $Inn(G)$ the set of all inner automorphisms of **G** is anormal subgroup of $Aut(G)$, the group of all automorphisms of **G**.

19. Define maximal normal subgroup of a group.Prove that **M** is a maximal normal subgroup of a group **G** if and only if the factor group G/M is simple.

20. Prove that every finite integral domain is a field

21. State and prove Fundamental homomorphism theorem for rings

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. Let G be a group with binary operation $*$ .Then prove the following:
    a) The left and right cancellation laws hold in G .
    b) The linear equations $a * x = b$ and $y * a = b$ have unique solutions x and y in G, where a and b are any elements of G .

23. State and prove **Cayley's theorem**. Give the elements for the left regular representation and the group table of the group given by the group table

	e	a	b
e	e	a	b
a	a	b	e
b	b	e	a

24. Let H be a subgroup of a group G. prove that $aHbH = abH$ defines a binary operation on G/H if and only if H is a normal subgroup of G. Then furthere show that if H is a normal subgroup of a group G then G/H is a group. under the binary operation $aHbH = abH$.

25. a) Prove that the divisors of 0 in Z_n are those nonzero elements that are not relatively prime to n .
    b) Find the divisors of Z_{16}
    c) Prove that Z_p , where p is prime has no divisors of 0.

(2×15=30)