QP CODE: 21100168

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) EXAMINATION, FEBRUARY 2021

Fifth Semester

Core Course - MM5CRT03 - ABSTRACT ALGEBRA

B.Sc Mathematics Model I,B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

9F863599

Time: 3 Hours | Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Write two examples for non structural property of a binary structure $\langle S, * \rangle$.
2. Define general linear group of degree n.
3. Define proper subgroup and improper subgroup of a group G .
4. Define a **permutation of a set**. Compute $\sigma\tau$ where σ and τ are permutations given by $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 5 & 3 & 1 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 4 & 2 & 1 \end{pmatrix}$.
5. Express the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 1 & 4 & 7 & 2 & 5 & 8 & 6 \end{pmatrix}$ in S_8 as a product of transpositions.
6. If $n \geq 2$, then prove that the collection of all even permutations of $\{1, 2, 3, \cdots, n\}$ forms a subgroup of the symmetric group S_n.
7. Prove that the order of an element of a finite group divides the order of the group.
8. Check whether $f : (M_n(\mathbb{R}), +) \to (\mathbb{R}, +)$ defined by $f(A) = det(A)$ is a group homomorphism or not.
9. Let G be a group and H be normal subgroup of G. Find the identity element in the factor group G/H.
10. Define a unit in a ring R , Find the units in Z_{10}
11. Define zero divisors . Find the divisors of Z_{12}

12. Define a factor ring .

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Let F be the set of all real-valued functions f having domain $\mathbb{R}$, the set of real numbers. Define addition, subtraction, multiplication and composition of such functions, and state whether F is closed under these operations.

14. a) Give the group table for the binary operation *addition modulo 2* on the set $\mathbb{Z}_2$.
b) Give the group table for a binary operation $*$ on the set $\{e, a, b\}$.

15. State and prove Division Algorithm for $\mathbb{Z}$.

16. Let G be a group. Prove that the permutations $\rho_a : G \to G,$ where $\rho_a(x) = xa$ for $a \in G$ and $x \in G$, do form a group isomorphic to G.

17. Let H be a subgroup of a group G. Let the relation $\sim_L$ be defined on G by $a \sim_L b$ if and only if $a^{-1}b \in H.$ Then show that $\sim_L$ is an equivalence relation on $G.$ What is the cell in the corresponding partition of G containing $a \in G$?

18. Let G be a group. Show that $Aut(G)$, the set of all automorphisms of G forms a group under function composition.

19. Obtain the converse statement of Lagranges theorem. Show that converse of Lagranges theorem is not true in general.

20. Check whether nZ with usual addition and multiplication is a ring.

21. Show that Ia = { $x \in R/ax = 0$ }is an ideal of R, R is a commutative ring and $a \in R$

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. Let G be a cyclic group with n elements and generated by a . Let $b \in G$ and let $b = a^s$. Then prove that b generates a cyclic subgroup H of G containing n/d elements, where d is the gcd of n and s .
Also prove that $\langle a^s \rangle = \langle a^t \rangle$ if and only if $gcd\ (s, n\ ) = gcd\ (t, n\ )$.

23. Prove in two methods (from linear algebra and by counting orbits) that no permutation in S_n can be expressed both as a product of an even number of transpositions and as a product of an odd number of transpositions.

24. State and prove fundamental homomorphism theorem.

25. a)Let R be commutative ring with unity of characteristic 4 , Compute and simplify $(a + b)^4$ for $a, b \in R$

b) Prove that every field F is an integral domain.

c) Prove that every finite integral domain is a field

(2×15=30)