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QP CODE: 20000643

Reg No : ....

Name : ....

MSc DEGREE (CSS) EXAMINATION , NOVEMBER 2020

Second Semester

CORE - ME010202 - ADVANCED TOPOLOGY

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 Admission Onwards

E4AE566B

Time: 3 Hours | Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Let X be a T_2 space and $x \in X$. Let F be a finite subset of X not containing x. Show that there exist open sets U and V in X such that $x \in U$, $F \subseteq V$ and $U \cap V = \emptyset$
2. Show that every compact Hausdorff space is normal
3. Define the topological product of the family of spaces $\{(X_i, \tau_i); i \in I\}$
4. Define a cube and a Hilbert cube
5. Explain the terms productive property, countably productive property and finitely productive property.
6. Let $f_1, f_2, f_3 : R \to R$ be defined by $f_1(x) = cosx, , f_2(x) = sinx, f_3(x) = x$ for $x \in R$. Describe the evaluation maps of the families $\{f_1, f_2\}, \{f_1, f_2, f_3\}, \{f_1, f_3\}$.
7. Give an example of a metric space which is not second countable.
8. Show that a first countable, countably compact space is sequentially compact.
9. Define the subnet of a net
10. When we say that two continuos functions f and g are homotopic?

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Let $F, G : X \to R$be two extensions of a continuous function f. Let $C = \{x \in X : F(x) = G(x)\}$.Then prove that C is a closed set

12. Let A be a closed subset of a normal space X and suppose f : A→[-1,1] is a continuous function. Show that there exists a continuous extension of f to X

13. 1) Define the cartesian product of a finite family of sets and extend this definition to an uncountable family of sets
2) Define the projection function and find the projection of the unit square $[0,1] \times [0,1]$ in the cordinate spaces

14. Prove that a subset of X is a box if and only if it is the intersection of a family of walls.

15. Prove that every pseudo-metric space is completely regular.

16. Prove that countable compactness is preserved under continuous functions.

17. Describe the following
1) Reimann Net using partitions of [0,1] as a directed set with a suitable follows relation
2) Let X be any set and $\mathcal{F}$ be the set of all finite subsets of X. For $F, G \in \mathcal{F}$ define $F \geq G$ to mean $F \supset G$. Prove that $\geq$ directs $\mathcal{F}$

18. Let X be the topological product of a family of spaces $\{ X_i : i \in I \}$.Prove that a net S : D → X converges to a point x ∈X iff for each i ∈ I the net $\pi_i \circ s$ converges to $\pi_i(x)$in X_i

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. Prove that a topological space X is normal if and only if two disjoint closed subsets A and B of X can be separated by means of a continuos real valued function f from X to [0,1] in the sense that f(A)=0 and f(B)=1

20. (a) Prove that a product of spaces is connected if and only if each coordinate space is connected
(b) Prove that a product of topological spaces is completely regular if and only if each coordinate space is so.

21. a) Explain the terms distinguish points and evaluation function. Characterise one-to one evaluation function.
b) If X is a Tychonoff space then prove that the family of all continuous real valued functions on X distinguishes points.

22. Define limit of a net in a topological space X. Prove that a topological space is Hausdroff iff limits of all nets in it are unique.

(2×5=10 weightage)