QP CODE: 21100166

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Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) EXAMINATION, FEBRUARY 2021

Fifth Semester

Core Course - MM5CRT01 - MATHEMATICAL ANALYSIS

B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science, B.Sc Computer Applications Model III Triple Main

2017 Admission Onwards

C9A38860

Time: 3 Hours | Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. State ordering property of real numbers? Is the set of all rational numbers ordered? Justify.
2. Find all $x \in R$ such that $|x-1| > |x+1|$?
3. Does $f(x) \leq g(x)\ \forall x \in D$ imply that $Sup\ f(D) \leq Inf\ g(D)$? Give proper reasoning?
4. Define Nested intervals? Is the interval $I_n = \left(-\frac{1}{n}, \frac{1}{n}\right), n \in N$ nested?
5. Prove that the sequence $(x_n) = ((-1)^n)$ does not converge.
6. Prove that a convergent sequence of real numbers is bounded.
7. Find $lim(\frac{n+1}{n\sqrt{n}})$.
8. Using Monotone Convergence Theorem, prove that $lim(\frac{1}{\sqrt{n}}) = 0$.
9. Use the recurrance relation of n^{th} term of a sequence that converges to $\sqrt{a}$ to find the value of $\sqrt{5}$ correct to 4 decimal places.
10. State Abel's Lemma.
11. Test the convergence of $\sum_1^\infty \frac{(-1)^{n+1}}{(n^2+1)}$
12. Show that $\lim_{x\to\infty} x^n = \infty$ for $n \in \mathcal{N}$.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Prove that If A, B are bounded sets then $Sup\ (A+B) = Sup\ A + Sup\ B$ where $A + B = \{a + b : a \in A, b \in B\}$

14. Prove that $x \in [0,1]$ then the binary representation of x forms a sequnce consisting only $0, 1$?

15. If c > 0, prove that lim $(c^{1/n}) = 1$.

16. Prove that (sin n) is divergent.

17. Let (x_n) and (y_n) be two sequences of real numbers and suppose that $x_n \leq y_n$ for all n. Prove that

(a) if lim $x_n = +\infty$ then lim $y_n = +\infty$.

(b) if lim $y_n = -\infty$ then lim $x_n = -\infty$.

18. Prove that the geometric series $\sum_{n=0}^{n=\infty} r^n$ converges if and only if $|r| < 1$.

19. If (a_n) is a decreasing sequence of strictly positive numbers and if $\sum a_n$ is convergent, show that $\lim na_n = 0$.

20. Show that $\lim_{x \to 0} sgn(x)$ does not exist.

21. Let $A \subseteq \mathscr{R}$, $f, g : A \to \mathscr{R}$, $c \in \mathscr{R}$ be a cluster point of A. If $f(x) \leq g(x)$ for all $x \in A, x \neq c$,Then prove the following

(a) If $\lim_{x \to c} f = \infty$, then $\lim_{x \to c} g = \infty$.

(b) If $\lim_{x \to c} g = -\infty$, then $\lim_{x \to c} f = -\infty$.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. Prove the denumerability of the following sets

(a.) The set of all rational numbers Q

(b) The set $N \times N$, where N is the set of all natural numbers

23. (a) State and prove Cauchy Convergence Criterion.

(b) Let X = (x_n) be the sequence defined as $x_1 = 1$, $x_2 = 2$ and $x_n = \frac{x_{n-2} + x_{n-1}}{2}$ for n > 2. Prove that lim X = $\frac{5}{3}$.

24. State and prove Raabe's test. Use this test to study the convergence of $\sum_1^\infty (\frac{n}{(n^2+1)})$.

25. (a) Let $A \subseteq \mathscr{R}$, $f : A \to \mathscr{R}$ and let $c \in \mathscr{R}$ be a cluster point of A. If $a \leq f(x) \leq b$ for all $x \in A$, $x \neq c$, and if $\lim_{x \to c} f$ exists,Then prove that $a \leq \lim_{x \to c} f \leq b$.

(b) Check whether the following limits exist or not. Give explanations

(1) $\lim_{x \to 0} cos(\frac{1}{x})$ (2) $\lim_{x \to 0} x cos(\frac{1}{x})$

(2×15=30)